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The survival of fossil magnetic fields during pre-main sequence evolution

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Abstract. The origin of the strong large-scale magnetic fields found at the surfaces of the near-main sequence chemically peculiar (CP) stars is still a matter of controversy. The fossil theory, in which the fields are explained as remnants of fields present during earlier stages of stellar evolution, arguably is better able to explain the observed CP star magnetism. However the question of whether significant large-scale magnetic flux can survive through the pre-main sequence evolution has been much disputed, but little explored. Here we attempt to make some preliminary, semi-quantitative estimates related to flux survival in the presence of large-scale convection. We also present a simple model that attempts to quantify the fraction of flux that can survive from the top of the Hayashi track to the main sequence. A broad conclusion is that for plausible values of parameters such as turbulent diffusivity, flux can more readily survive in stars of several, rather than about one, solar masses, although the contrast is not as strong as appears to be implied by observations. Attention is drawn to the effects of uncertainties in modelling pre-main sequence stellar evolution.

Key words. magnetic fields – magnetohydrodynamics (MHD) – stars: evolution – stars: chemically peculiar

1. Introduction

In a radiative star, the resistive decay time of a large-scale magnetic field is $\sim R^2/\eta$, where η is the electrical resistivity. A priori, this is of order of, or maybe considerably longer than, a star's main sequence lifetime (e.g. Cowling 1945). This observation has led to the suggestion that large-scale magnetic fields may, in suitable circumstances, survive with little loss of net (signed) flux for astrophysically significant times. In particular, it has been proposed that the anomalously strong magnetic fields at the surfaces of the near-main sequence CP stars are relics of the field present in the interstellar medium from which they formed (e.g. Cowling 1945). This "fossil field" theory for the CP star fields appears capable of avoiding many of the difficulties encountered by the rival contemporary core-dynamo theory (see e.g. Moss 1989, 2001; Mestel 1999). In particular the "10% problem" – that only about 10% of near-main sequence stars in the CP star mass range display strong large-scale fields at their surfaces – is primarily explained by variations in the amount of magnetic flux trapped during star formation, with only the high flux tail of the distribution resulting in stars that are observably magnetic on the main sequence.

However, one important issue has been rather skirted around when developing these ideas. The CP stars, of several solar masses, are usually believed to have undergone an epoch of largely or fully convective pre-main sequence evolution, as they descend their Hayashi tracks (e.g. Hayashi et al. 1962; Iben 1965; and many other authors). If magnetic fields are dynamically unimportant, the turbulent motions will tangle a field to a scale at least as small as l, the typical length scale of the turbulence, thus greatly accelerating the field decay. This process is conveniently, if rather naively, parameterized by the introduction of a turbulent diffusivity $\eta_T = kv_t l$, where $k = O(1)$ and v_t is a typical turbulent velocity (conventionally $k = 1/3$). With conventional estimates for v_t and l, decay times during pre-main sequence evolution are $t_D \sim 10^4$ yr, and thus fossil fields would disappear long before the main sequence was reached. However, magnetic fields are not necessarily passive, and fields of about equipartition strength or more, $|\boldsymbol{B}| \gtrsim B_{eq} = \sqrt{4\pi\rho}v_t$, will strongly modify the turbulent motions, inhibiting field tangling and subsequent decay. This is especially true if the fields are at least partly helical (e.g. Blackman & Brandenburg 2002). A plausible scenario, given general support by numerical simulations, is that the field can become concentrated into ropes of equipartition strength or greater. The fluid motions are strongly inhibited in and near the ropes, but continue more-or-less freely between them (e.g. Weiss et al. 2002). In the language of mean field electrodynamics, a strong η-quenching occurs when $|\boldsymbol{B}| \gtrsim B_{eq}$ (e.g. Rogachevskii & Kleeorin 2001; see also, e.g., Brandenburg & Sokoloff 2002), although this may not be a very appropriate description at small or intermediate scales. There appear to be no numerical simulations directly relevant to the situation in question, but below we attempt to make some rather crude, order of magnitude, estimates.

If a radiative core develops, the field lines passing through the core will be more-or-less frozen into the material, and thus immune to further turbulent distortion and decay, even if their extensions beyond the core into the radiative envelope continue

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to be distorted. We present in Sect. 3 a preliminary attempt to quantify this process.

An important complicating factor is dynamo action, which may well occur in these convecting and rotating objects. The interaction of a pre-existing dynamically important field with a dynamo has been little explored – some speculations are made in Sect. 4.

It should be noted here that Palla & Stahler (1993 and references therein) propose a rather different picture of pre-main sequence evolution. Starting from non-standard initial conditions, designed to be consistent with the preceding dynamical collapse of the pre-stellar material, they found that stars with masses $M \gtrsim 2.5\ M_\odot$ (this mass depending somewhat on details of the model) always have at least a radiative core during pre-main sequence evolution. If their picture is substantially valid, then many of the difficulties discussed in this paper will be much alleviated.

2. Magnetic field strengths

2.1. Pre-stellar fields

The collapse of a gas cloud from initial number density $n \sim 300$ to $n \sim 10^{12}$ has been followed by e.g., Desch & Mouschovias (2001). Nakano et al. (2002) have performed comparable simulations, making some rather different assumptions but with similar conclusions concerning the field strengths and gas densities at recoupling. In brief, in their standard case the cloud collapses maintaining a high enough degree of ionization to conserve its magnetic flux until $n \sim 10^{10}$–10^{11} cm^{-3}. With an assumed initial field at $n \sim 300$ of 30 μG, the central field strength is then about 0.1 G. The cloud then becomes opaque, the level of ionization decreases, and the field decouples from the matter. The collapse proceeds adiabatically, and at $n \sim 10^{15}$–10^{16} cm^{-3} the temperature has risen enough to increase the degree of ionization so that field freezing again occurs, with the field strength still about 0.1 G. This field strength appears not to depend sensitively on the mass of the cloud. A further dynamical stage of adiabatic collapse occurs, conserving magnetic flux until the pre-stellar object arrives at the top of the Hayashi track. Here the radius is $R_{\rm i}$, and a value of the corresponding field strength $B_{\rm i}$ can be estimated to order of magnitude from the ratio of the density there to the density at recoupling (by, for example, assuming isotropic collapse). We describe the subsequent stellar evolution by the models of Siess et al. (2000), considering explicitly models of 1, 3 and 5 $M_\odot$, with $B_{\rm i} \sim 250, 200, 40$ G respectively.

Inevitably there are considerable uncertainties connected with this scenario, but it does avoid almost total arbitrariness in the estimate of $B_{\rm i}$. In this spirit, we ignore any spatial variation of field strength. In particular note that, among other factors, the initial ISM field strength might be rather larger or smaller than the value adopted, with proportional effects on $B_{\rm i}$ and consequent fields – see below.

2.2. Estimation of B_{eq}

The value of $B_{\rm eq}$ near the centre of a fully convective star can be estimated using the simple form of mixing length theory presented by Schwarzschild (1958). His Eqs. (7.4) and (7.5) are

$$\Delta\nabla T = \frac{L(r)}{4\pi r^2}\frac{1}{(c_P \rho v_{\rm t} l)}, \qquad (1)$$

and

$$v_{\rm t} = (\Delta\nabla T g/T)^{1/2} l, \qquad (2)$$

in a standard notation. From these, and the mass continuity equation

$$\frac{{\rm d}M(r)}{{\rm d}r} = 4\pi\rho r^2, \qquad (3)$$

and the form of the energy generation equation applicable to gravitationally contracting stars with no nuclear energy sources,

$$\frac{{\rm d}L(r)}{{\rm d}r} = 4\pi\rho r^2 \epsilon_{\rm g}, \qquad (4)$$

where

$$\epsilon_{\rm g} = \frac{3}{2}\frac{P}{\rho}J \qquad (5)$$

and $J = -\frac{1}{R}\frac{{\rm d}R}{{\rm d}t}$, we obtain

$$B_{\rm eq} \sim 0.014\rho_{\rm c}^{5/6} J^{1/3} (r^2 l)^{1/3}\ \ {\rm G}. \qquad (6)$$

Here $\rho_{\rm c}$ is the central density, r is the local radius, R is the stellar radius, and the estimate for $B_{\rm eq}$ holds near the centre of the star.

2.3. Field strengths on the Hayashi track

Assuming flux conservation, at time t and radius $R(t)$ the mean field strength is $B(t) = (\frac{R_{\rm i}}{R})^2 B_{\rm i}$. If $B(t) \gtrsim B_{\rm eq}$, turbulent decay will be strongly inhibited throughout the star. However, this is likely to be an unrealistically (and unnecessarily) strong condition. If the field is weaker than $B_{\rm eq}$, the turbulent motions will tangle and reduce the scale of the field, simultaneously increasing the field strength. If the field reaches equipartition strength locally at scale $d \gtrsim l$, then further tangling will be resisted in the vicinity of the resulting field ropes. The field strength in the ropes will be $B_{\rm rope} \approx B_{\rm i}(\frac{R_{\rm i}}{d})^2$ (this argument is due to Tayler 1987). The maximum value $B_{\rm rom}$ of $B_{\rm rope}$ is given when $d = l$; if this value is smaller then $B_{\rm eq}$ then turbulence enhanced decay cannot be avoided. Eventually, as the star moves down the Hayashi track in the HR diagram, a radiative core forms. We assume that any flux threading this inner radiative region will thereafter be effectively immune to turbulent decay, even if the parts of these ropes that penetrate the convective outer layers continue to be deformed. We also assume that any topological readjustment necessary to produce a dynamically stable configuration, e.g. with linked poloidal and toroidal fluxes, will take place (see Wright 1973; Markey & Tayler 1973, 1974; and Chap. 5 of Mestel 1999).

Table 1. The asterisked entries indicate the oldest models before a radiative core develops. The third column gives the fractional radius of the radiative core for the later models.

t(yr)	$R/R_\odot$	r_c/R	B_{eq}(G)	B_{rom} (G)	$B(t)$ (G)
$M = 1\ M_\odot$, $B_i = 250$ G					
10^3	13	0.0	2.5×10^3	2×10^4	250
2×10^5	6	0.0	2×10^3	3.5×10^4	10^3
5×10^5	3	0.0	4×10^4	1.7×10^5	4×10^3
$2 \times 10^{6*}$	2	0.0	1.4×10^4	3.8×10^5	10^4
3×10^6	1.8	0.08	7.8×10^3	4.7×10^5	1.3×10^4
$M = 3\ M_\odot$, $B_i = 200$ G					
10^3	22	0.0	10^3	4.7×10^4	210
5.3×10^4	15	0.0	850	4.7×10^4	430
1.1×10^5	13	0.0	1.5×10^3	4.7×10^4	600
$1.5 \times 10^{5*}$	11	0.0	3×10^3	4.7×10^4	840
2×10^5	9	0.25	3.8×10^3	4.7×10^4	1.2×10^3
$M = 5\ M_\odot$, $B_i = 40$ G					
0	45	0.0	340	4.0×10^4	50
8.8×10^3	30.5	0.0	670	4.0×10^4	90
5.9×10^4	21.6	0.0	630	4.0×10^4	170
$9.6 \times 10^{4*}$	19.3	0.0	1.1×10^3	4.0×10^4	220
1.1×10^5	18.0	0.42	1.6×10^3	4.0×10^4	260

We give in Table 1 values of B_{eq} (Eq. (6)), $B(t)$ and B_{rom} for masses 1, 3, 5 $M_\odot$, estimating the stellar parameters from Siess et al. (2000). In this argument a major source of uncertainty is the value of the mixing length near the centre of a convecting sphere where, e.g., the pressure scale height formally becomes very large, but the convective flux becomes small especially if, as considered here, the energy flux results predominantly from the release of energy by gravitational contraction. In Table 1 we have rather arbitrarily used $l = r = \min(R/6, 10^{11}$ cm). In one way these arguments are deliberately conservative, in that any field amplification from rotational shear has been ignored.

We see that $B(t) < B_{eq}$ in all cases. However B_{rom} exceeds B_{eq} by a substantial factor. These results suggest that dynamically important flux ropes may form in stars of all the masses considered. The only distinguishing feature between the entries in Table 1 appears to be the time before the radiative core forms, and perhaps the rate at which it grows. This prompts the suggestion that even when dynamically important flux ropes form, diffusive decay continues, but at a much reduced rate. We now develop a simple numerical model to illustrate the possible importance of these factors.

3. The model

We consider a sphere of radius $R(t)$, and solve the induction equation

$$\frac{d\boldsymbol{B}}{dt} = -\nabla \times \eta \nabla \times \boldsymbol{B}, \tag{7}$$

where d/dt is the Lagrangian derivative following the radial inflow as the star contracts. The radius $R(t)$ is taken from the evolutionary models of Siess et al. (2000) for a range of stellar masses M. These models also provide the radius $r_c(t)$ of the radiative core, and the times t_c, t_{MS} at which the core forms, and at which the main sequence is reached.

We consider an axisymmetric poloidal field

$$\boldsymbol{B} = \nabla \times \left(\frac{\psi}{r \sin\theta} \hat{\phi} \right) \tag{8}$$

only, so ψ is a "streamfunction" for the field. When η is independent of θ, then if we take the simplest large-scale structure for $\boldsymbol{B}$, writing $\psi(r, \theta) = b(r) \sin^2\theta$, we obtain

$$\frac{db}{dt} = \eta \left(\frac{\partial^2 b}{\partial r^2} - \frac{2b}{r^2} \right), \tag{9}$$

and the angular structure is conserved during subsequent evolution.

The resistance of the dynamically important flux ropes to further tangling and decay is modelled by introducing a naive η-quenching,

$$\eta = \frac{\eta_0}{1 + (\boldsymbol{B}^2/B_{eq}^2)}, \tag{10}$$

where $B_{eq}^2 = 4\pi\rho v_t^2$ defines the equipartition energy, v_t being a typical turbulent velocity. In order to keep the simple θ-independent structure of Eq. (9), we rewrite Eq. (10) as

$$\eta = \frac{\eta_0}{1 + (< \boldsymbol{B}^2 > /B_{eq}^2)}, \tag{11}$$

the angular brackets denoting an average over a shell of radius r. This expression is used in $r > r_c(t)$. In $r \le r_c(t)$, $\eta = 0$. Eq. (9) is solved for a variety of masses M, unquenched diffusivities η_0 and initial values $\beta = |\boldsymbol{B}|/B_{eq}$, to find the fraction γ of the initial flux that remains at time $t = t_{MS}$.

The initial field is chosen to be parallel to the axis $\theta = 0$, and is either uniform, $b(r) \propto r^2$, or mimics the mildly centrally condensed field in a $n = 1.5$ polytrope that has contracted isotropically from an initially uniform state, so $B \propto \rho^{2/3}$ in the equatorial plane. The results for the two cases are quite similar, and only the latter will be discussed here. The dependence of γ on M, $1\ M_\odot \le M \le 5\ M_\odot$, is shown in Fig. 1 for several values of η_0 and β. As a rule of thumb, simultaneously increasing β by a factor of 3 and η_0 by a factor 10 leaves results approximately unchanged, as might be anticipated from Eq. (11) for $B/B_{eq} > 1$ (in this figure, two of the curves show the approximate results for pairs of parameter values that yield very similar behaviour). In Fig. 2, γ is shown as a function of η_0, for $\beta = 0.77$ and 2.3. Figure 3 shows γ as a function of β for $\eta_0 = 10^{12}$ cm^2 s^{-1} and 10^{13} cm^2 s^{-1}.

4. Discussion

From Figs. 1 and 2 it is clear that the flux survival fraction γ decreases as M decreases, and that for $\eta_0 \gtrsim 10^{12}$ cm^2 s^{-1} there is significant flux survival only for $\beta > 1$. Major uncertainties are, of course, the appropriate values to take for η_0 and β.

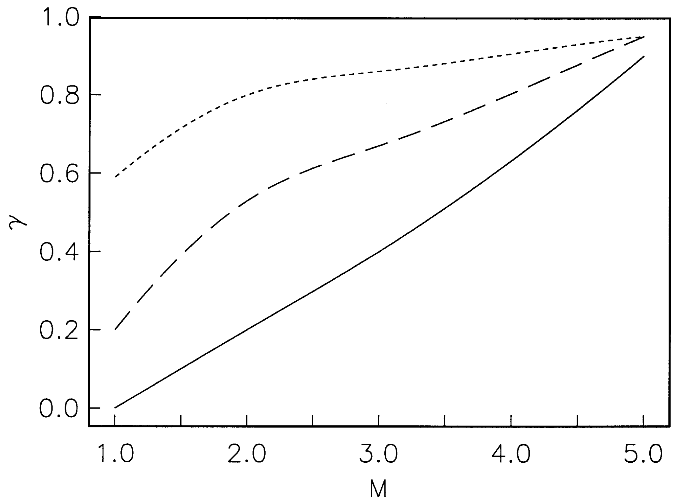

Fig. 1. The variation of the fraction of initial flux $\gamma = \psi(\mathrm{MS})/\psi(0)$ that survives to the main sequence, as a function of mass (in units of $M_\odot$) for $\eta_0 = 10^{12}$ cm^2 s^{-1}, $\beta = 4.6$ (short-dashed); $\eta_0 = 10^{11}$ cm^2 s^{-1}, $\beta = 0.77/\eta_0 = 10^{12}$ cm^2 s^{-1}, $\beta = 2.3$ (long-dashed); $\eta_0 = 10^{13}$ cm^2 s^{-1}, $\beta = 4.6/\eta_0 = 10^{12}$ cm^2 s^{-1}, $\beta = 1.5$ (solid).

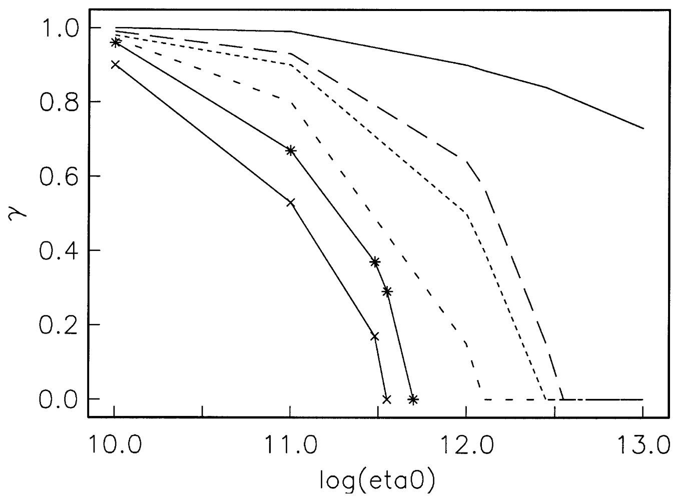

Fig. 2. The variation of γ, the fraction of initial flux that survives to the main sequence, as a function of η_0 when $\beta = 2.3$ for $M = 5\,M_\odot, 3\,M_\odot, 2\,M_\odot, 1\,M_\odot$, respectively solid, long-dashed, short-dashed and long-spaced curves. Also results for $M = 3\,M_\odot$ and $2\,M_\odot$ with $\beta = 0.77$ are shown by the solid curves joining asterisks and crosses respectively.

Naive mixing length theory estimates in the present day Sun are $v_\mathrm{t} \sim 10^4$ cm s^{-1}, $l \sim$ few $\times\,10^9$ cm, giving $\eta_T \sim 10^{13}$ cm^2 s^{-1}. Dynamo theory tends to favour rather smaller values of η_T, on rather pragmatic grounds, say $(3\times10^{11}$–$3\times10^{12})$ cm^2 s^{-1}. We are here considering a rather different phase of stellar evolution, but 10^{11} cm^2 s^{-1} $\le \eta_T \le 10^{13}$ cm^2 s^{-1} seems a reasonable range. In the context of this model, β must be a proxy for the ability of the field in ropes to resist distortion and decay – see e.g. Sect. 2.3 – and probably it is not reasonable here to say more than that.

Clearly γ is also a sensitive function of β – see Fig. 3. For example, in order to have $\gamma \gtrsim 0.2$ with $M \gtrsim 2\,M_\odot$ and $\eta_0 = 10^{13}$ cm^2 s^{-1}, $\log\beta \gtrsim 0.7$. If $\eta_0 = 10^{12}$ cm^2 s^{-1}, $\gamma \gtrsim 0.2$ requires $\log\beta \gtrsim 0.2$.

Thus, this simple-minded experiment provides some support for the proposal that more massive stars retain more of their initial flux, and that stars with stronger than average initial fields are likely to be more strongly magnetic on the main sequence. Interestingly, the physically plausible ranges of the parameters η_0 and, more uncertainly, β do appear to cover the region of parameter space where essentially zero flux survival changes to substantial survival. However, there is no steep cutoff either with mass or β, which is perhaps a little disappointing.

The interaction of a strong fossil flux with a dynamo generated field has been largely ignored in this context (but see Moss & Shukurov 2001 for computations relevant to a related situation in spiral galaxies), even though rotating convecting regions are prime candidates for sites of dynamos (if the angular velocity is large enough). If the differential rotation is small, and the dynamo is approximately of α^2 type, then simple dynamo models suggest that dynamically strong nonaxisymmetric seed fields may be "remembered" for as long as $O(100)$ diffusion times (with $t_\mathrm{D} \sim 10^4$ yr – e.g. Rädler et al. 1990; Moss et al. 1991). In general such transient fields have an approximate axis of symmetry aligned at angle χ to the rotation axis that is neither zero nor $\pi/2$, even though the final stable states generally have $\chi = 0$ or $\pi/2$ (e.g. Rädler et al. 1990; Moss et al. 1991, 1995). Any such field may become frozen into the growing radiative core, and its "fossilized" nonaxisymmetric structure thus preserved. Such a field has been called a "hybrid" field.

5. Conclusions

Order of magnitude estimates, together with simple-minded numerical experiments, provide some support for the idea that significantly more primeval magnetic flux may survive to the main sequence in middle main sequence than lower main sequence ($M \lesssim 1\,M_\odot$) stars. However the models suggest a smooth transition with increasing mass, rather than a threshold mass. If it were not for the example of the Sun, it might be argued that the "extra factor" distinguishing magnetic stars of several solar masses from the Sun (and other lower mass stars) is the presence of a significant convective envelope in lower mass stars on the main sequence, which serves to bury any surface field and so hide fossil flux from observation; however there is evidence for at most a small fossil field in the solar interior (Pudvokin & Benevolenskaya 1984; Boyer & Levy 1984; Boruta 1996). Given the naivety of the ideas explored above, perhaps this unresolved situation is not so surprising. It does suggest that the problem may be worth further attention; however significant progress may need input from large-scale MHD simulations.

Finally, it should be remarked again that if the scenario of Palla & Stahler (1993) is valid, then only stars with $M \lesssim 2\,M_\odot$ undergo a substantial interval of fully convective pre-main sequence evolution. Thus fossil fields in more massive stars would neither suffer enhanced decay, nor would there be the complicating possibility of pre-main sequence turbulent

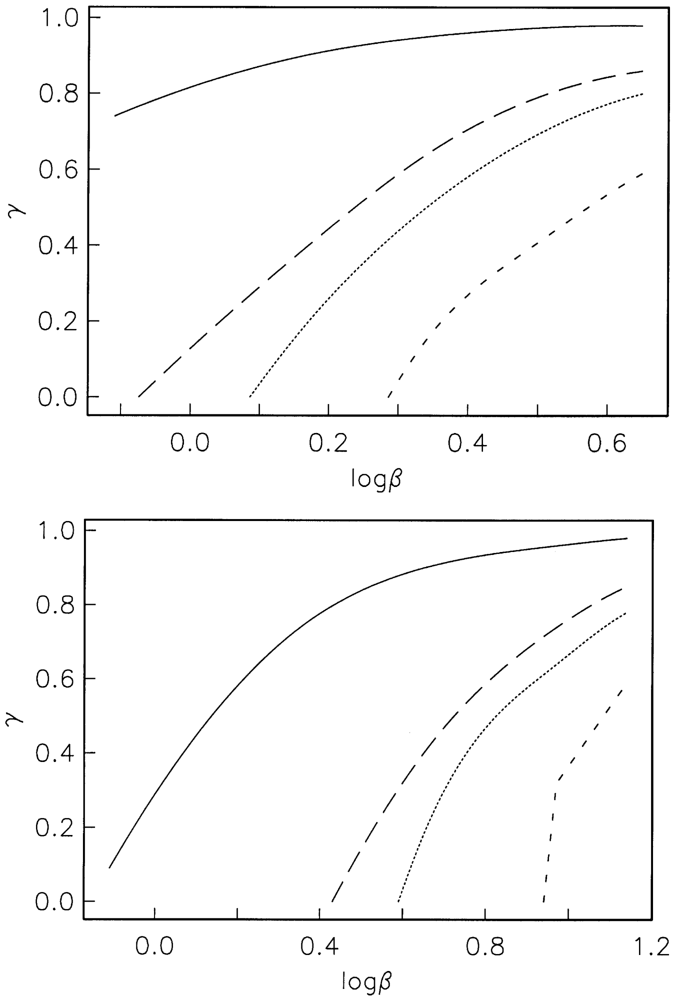

Fig. 3. The variation of the fraction of initial flux that survives to the main sequence as a function of initial field strength β for $\eta_0 = 10^{12}$ cm^2 s^{-1} (*upper panel*) and $\eta_0 = 10^{13}$ cm^2 s^{-1} (*lower panel*). In each plot the results are for $M = 5, 3, 2, 1\ M_\odot$ (solid, long-dashed, short-dashed and long-spaced curves respectively).

dynamo action. There would then be a clear watershed in mass between stars that appear to be readily able to retain primordial magnetic flux and those for which there are, at least apparent, difficulties (see also Stepien 2000).

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