

Period change and stellar evolution of β Cephei stars

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ABSTRACT

The β Cephei stars represent an important class of massive star pulsators that probe the evolution of B-type stars and the transition from main sequence to hydrogen-shell burning evolution. By understanding β Cep stars, we gain insights into the detailed physics of massive star evolution, including rotational mixing, convective core overshooting, magnetic fields, and stellar winds, all of which play important roles. Similarly, modeling their pulsation provides additional information into their interior structures. Furthermore, measurements of the rate of change of pulsation period offer a direct measure of β Cephei stellar evolution. In this work, we compute state-of-the-art stellar evolution models assuming different amounts of initial rotation and convective core overshoot and measure the theoretical rates of period change, that we compare to rates previously measured for a sample of β Cephei stars. The results of this comparison are mixed. For three stars, the rates are too low to infer any information from stellar evolution models, whereas for three other stars the rates are too high. We infer stellar parameters, such as mass and age, for two β Cephei stars: ξ^1 CMa and δ Cet, which agree well with independent measurements. We explore ideas for why models may not predict the higher rates of period change. In particular, period drifts in β Cep stars can artificially lead to overestimated rates of secular period change.

Key words. stars: evolution – stars: fundamental parameters – stars: mass-loss – stars: rotation

1. Introduction

Massive B-type stars are powerful engines that drive cosmic evolution from star formation through galaxy evolution and chemical enrichment, and to the seeds of life in the Universe. These stars affect star formation, through their winds contributing momentum and turbulence into the interstellar medium, and chemically enrich the ISM through their supernovae. Exploring the physics of these massive stars contributes to the understanding of cosmic evolution, as well as of stellar physics in general. Massive stars are also important probes of the physics of stellar evolution because these B-type stars display many different kinds of phenomena related to the physics of stellar rotation, stellar winds, pulsation, binarity, magnetic fields, and possibly stellar mergers (Langer 2012). For instance, the emission line stars display rapid, nearly critical, rotation, while the β Cephei stars undergo radial and non-radial pulsation (Townsend et al. 2004; Moskalik & Dziembowski 1992). In this work, we explore the β Cephei variable stars because their pulsation allows us to study their internal structure and potentially their evolution.

The β Cephei variable stars are particularly interesting because they display a diverse range of phenomena. Some β Cep stars appear to have strong magnetic fields (Silvester et al. 2009; Hubrig et al. 2011; Neiner et al. 2012) while others have weak magnetic fields or none at all (Silvester et al. 2009; Fossati et al. 2015); some rotate rapidly (Aerts et al. 1994; Handler et al. 2012), while others rotate slowly (Shultz et al. 2015). Most, if not all, β Cep stars tend to have weak stellar winds (Martins et al. 2005; Huenemoerder et al. 2012) and are also X-ray sources. Recently, Oskinova et al. (2014) have discovered X-ray variability in one β Cephei star ξ^1 CMa over a pulsation cycle. Understanding the physics in these stars influences our understanding of massive star physics in general and of the evolution

from the main sequence to the red supergiant stage of evolution, in particular.

While probing the physics of massive stars, the β Cephei stars also span the boundary between the main sequence evolution and the transition to the blue supergiant stage of evolution (Demarque & Percy 1965; Stothers 1965). As such, observations and models of these stars help us understand the transition and precisely the location of where on the Hertzsprung-Russell diagram it occurs thus, again, providing opportunities to refine the input physics in stellar evolution models.

In particular, there are two aspects of stellar physics that can be tested using models of β Cephei variable stars and observations: convective core overshooting and rotational mixing (Miglio et al. 2009; Lovekin & Goupil 2010). Convective core overshooting mixes material into the stellar core from layers that are some fraction of a pressure scale height above. In stellar evolution models, mixing-length theory treats convection as being fixed within the convective region such that the velocity and acceleration of a convective eddy goes to zero at the boundaries. Convective core overshooting is an ad hoc prescription in stellar evolution codes designed to account for the fact that convective eddies will not have zero velocity at the boundary between convective and radiative regions, hence penetrating above the convective core and mixing additional hydrogen into the stellar core. This overshooting acts to extend the main sequence lifetime of a massive star by creating a more massive core at the end of main sequence evolution. Rotational mixing will have similar effects on massive star evolution by meridional mixing (e.g., Frischknecht et al. 2010; Brott et al. 2011b).

Thanks to asteroseismic observations of β Cephei variables, there have been numerous measurements of the amount of convective core overshoot, parameterized as a fraction of a pressure scale height, $\alpha_{\text{co}}H_p$, as the total amount of material mixed

over a main sequence lifetime. The typical amount of convective core overshoot required to match observations is about $\alpha_{\text{co}} = 0.2$ and is consistent with measurements for the β Cep stars β CMA (Mazumdar et al. 2006) and δ Ceti (Aerts et al. 2006), as well as for classical Cepheids (Cassisi & Salaris 2011; Neilson et al. 2011; Neilson & Langer 2012; Prada Moroni et al. 2012) and other stars. However, measurements for other β Cep stars require different amounts of overshoot: θ Oph $\alpha_{\text{co}} = 0.4$ (Briquet et al. 2007) $0.28H_{\text{p}}$ (Lovekin & Goupil 2010), HD 129929 $\alpha_{\text{co}} = 0.1$ (Aerts et al. 2003), and ν Eridani $\alpha_{\text{co}} = 0.28$ (Suárez et al. 2009), among others. These measurements are obtained by comparing pulsation and evolutionary models of β Cep stars.

The degeneracy between rotational mixing and convective core overshooting is difficult to break. Rotational mixing can be constrained from measurements of the [N/C] enhancement (Hunter et al. 2008; Brott et al. 2011b), but this enhancement has been detected in some slowly rotating stars (Morel et al. 2006). There is no obvious way to constrain the importance of one process relative to another. The goal of this work is to compare new stellar evolution models with rotation to rates of period change measured for Galactic β Cephei stars and test whether period change is a probe of rotation and overshoot.

Eggleton & Percy (1973) first compared stellar evolution models with period change measurements and determined that the rate of period change is sensitive to the stage of evolution, be it main sequence or blue supergiant. However, since that work, stellar evolution models have matured, incorporating new opacities plus improved prescriptions for physics of stellar mass loss, rotation, and magnetic fields (Yoon & Langer 2005). For example, these changes have led to the realization that β Cephei pulsation is driven by the iron opacity bump (Moskalik & Dziembowski 1992; Cox et al. 1992; Dziembowski et al. 1993) as opposed to helium ionization in classical Cepheid and RR Lyrae stars. With the advances in stellar evolution models, it is an opportune time to revisit the comparison with period change measurements.

In the next section, we discuss the stellar evolution models used in this work and compute theoretical rates of pulsation period change in Sect. 3. In Sect. 4, we compare the rates of period change and test how period change relates to rotation rates. In Sect. 5, we discuss and summarize the results.

2. Models

We compute stellar evolution models using the Yoon & Langer (2005) code for masses ranging from $M = 7$ to $20 M_{\odot}$ in steps of one solar mass with various assumptions for internal mixing. One grid of models is computed assuming moderate convective core overshooting at $\alpha_{\text{co}} = 0.2$ (as described by for example Huang & Weigert 1983; Neilson et al. 2011) and with initial rotation rates $v_{\text{rot}} = 100, 200,$ and 400 km s^{-1} . We include rotational mixing in these models based on the prescription of Heger & Langer (2000), which is described in more detail by Brott et al. (2011a,b). A second grid of models is computed without rotation but assuming convective overshooting parameters $\alpha_{\text{co}} = 0, 0.2,$ and 0.4 .

The stellar evolution models are computed assuming radiatively driven winds as prescribed by Vink et al. (2000) during main sequence and blue supergiant evolution and with a Grevesse & Sauval (1998) solar metallicity. We compute the evolution models from the zero-age main sequence past the boundaries of the β Cephei instability strip as measured by Pamyatnykh (2007). Sample stellar evolution tracks are plotted

Table 1. Rates of period change for a sample of β Cep stars from Jerzykiewicz (1999).

Star	Period (d)	\dot{P}/P (Myr $^{-1}$)
γ Peg	0.15175	$+0.046 \pm 0.015$ or 0.0 ± 0.11
δ Cet	0.16114	$+0.20 \pm 0.01$ or 0.34 ± 0.06
ξ^1 CMA	0.20958	$+0.20 \pm 0.03$
α Lup	0.25985	$+0.84 \pm 0.22$
σ Sco	0.24684	$+1.55 \pm 0.14$
V2052 Oph	0.13989	-0.10 ± 0.22
BW Vul	0.20104	$+1.35 \pm 0.02$
β Cep	0.19049	$+0.06 \pm 0.06$

in Fig. 1 for different rotation rates and convective core overshooting parameters.

3. Period change

The purpose of this work is to compare measured rates of period change from observations with those predicted from stellar evolution models representing β Cephei stars. To do so, we use an analytic model of period change based on the period-mean density relation (Eddington 1918), $P \sqrt{\rho} = Q$, similar to that of Eggleton & Percy (1973). We do not use the derivations from Neilson et al. (2012a,b) because there is no evidence that the pulsation constant, Q varies as a function of period for β Cephei variables. Therefore, we write the relative rate of period change as

$$\frac{\dot{P}}{P} = -\frac{1}{2} \frac{\dot{M}}{M} + \frac{3}{2} \frac{\dot{R}}{R}, \quad (1)$$

where \dot{M} the rate of change of stellar mass. This relation measures the evolution of the star as a function of time.

We computed relative rates of period change from the stellar evolution models using this relation. Rates of period change are shown in Fig. 2 for two sets of models. The first set of models is computed assuming zero initial rotation, but with different amounts convective core overshooting, while the second set of models assume moderate convective core overshooting only but varying the initial rotation rates. This result suggests that the combination of measured rates of period change and a fundamental stellar parameter such as stellar radius, effective temperature or luminosity cannot provide a meaningful measure of physical processes such as convective core overshooting and/or rotational mixing (e.g., Miglio et al. 2009). These mixing processes act to change the mean structure of the star, but only in a way that is degenerate with models with different stellar masses but no additional mixing.

As part of this analysis, we use rates of period change presented previously by Jerzykiewicz & Pigulski (1999) and Jerzykiewicz (1999) for eight β Cephei stars, see Table 1. It should be noted that these rates differ from those presented by Eggleton & Percy (1973), based on different analyses. Measuring period changes for β Cephei stars is a challenge as for many of these stars there are multiple oscillation frequencies and the periods might confuse the measurement. A second issue has been addressed by Pigulski (1992, 1993, 1994), for which binary companions affect measurements of period change due to the light time effect. While this is not a complete sample of period change measurements for β Cephei stars, it does represent a well-measured sample.

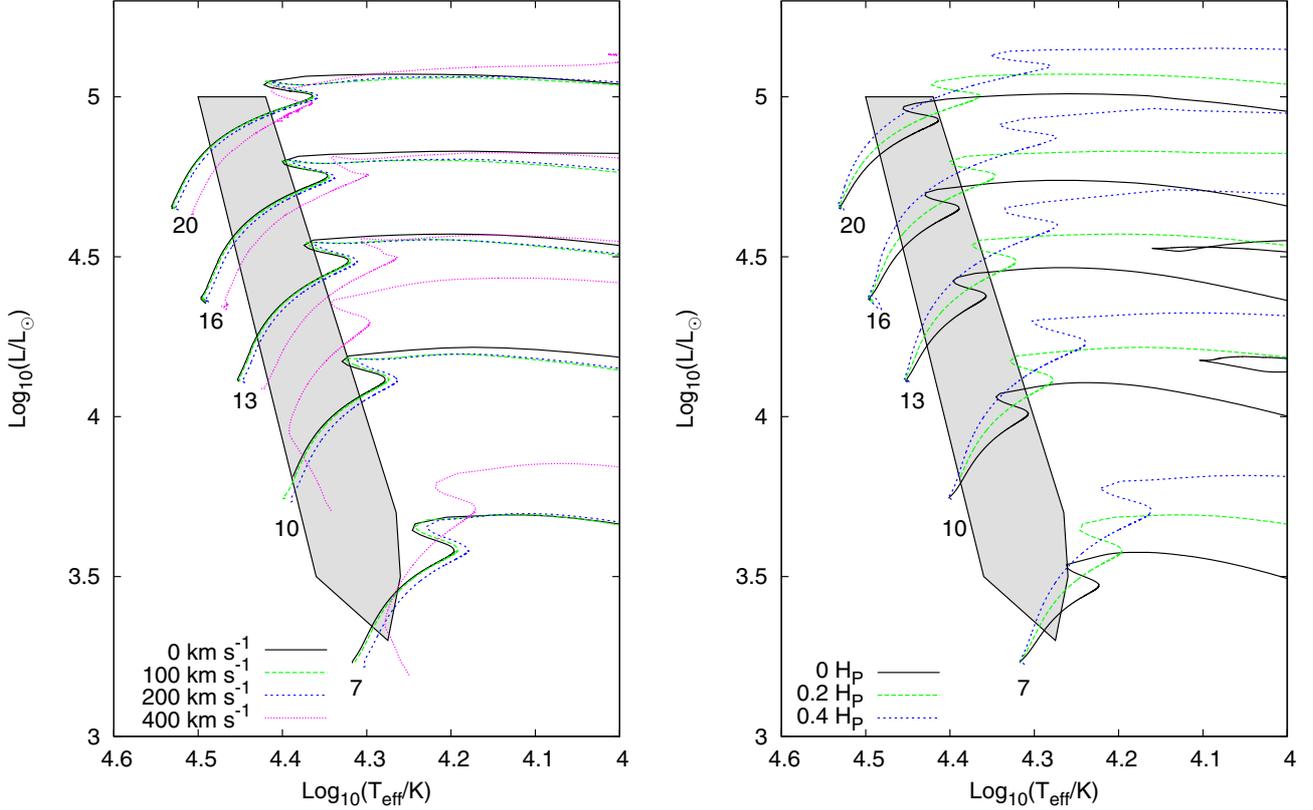


Fig. 1. *Left:* stellar evolution tracks assuming moderate convective core overshoot, $\alpha_c = 0.2$ and initial rotation rates of 0, 100, 200, and 400 km s^{-1} denoted by black solid, green dashed, blue dot-dashed, and magenta dotted lines, respectively. *Right:* stellar evolution tracks assuming zero rotation and convective core overshoot, $\alpha_c = 0, 0.2,$ and 0.4 as denoted by black solid, green dashed, and blue dot-dashed lines, respectively.

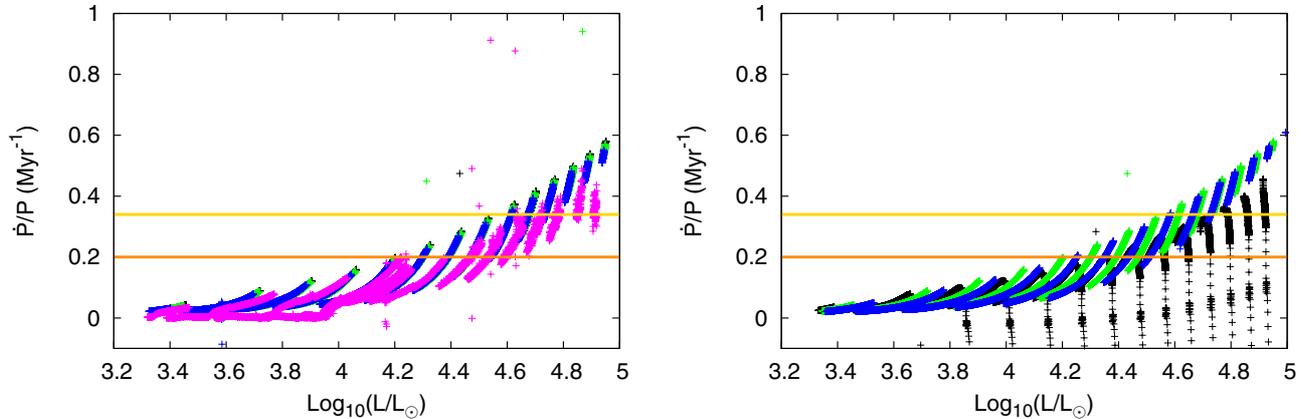


Fig. 2. *Left:* rates of period change computed from stellar evolution tracks assuming moderate convective core overshoot, $\alpha_c = 0.2$ and initial rotation rates of 0, 100, 200, and 400 km s^{-1} . *Right:* rates of period change computed from stellar evolution tracks assuming zero rotation and convective core overshoot, $\alpha_c = 0, 0.2,$ and 0.4 . The different colors are the same as for Fig. 1 while the horizontal orange line denotes the measured rates of period change for ξ^1 CMa and δ Cet and the gold line denotes the second measured value for δ Cet.

4. Results

We compare theoretical and measured rates of period changes for the sample of β Cep stars in Table 1. From these measurements, we can compute stellar fundamental parameters for models with rates of period change that is consistent with each observation. Based on the rates presented in Table 1 and the models plotting in Fig. 2, we break the comparisons into three categories: 1) period change rates fit by almost all models; 2) period change rates fit by specific models; and 3) period change rates fit by *no* models. The first category simply suggests that the measured rate of period change offers no constraints on the

models, while the third category suggests that the period change is completely inconsistent with models. These differences appear to be a result of insufficient precision for some observations and that observations also span a greater range of period changes rates than predicted from the stellar evolution models.

There are three β Cep stars for which the measured rates of period change offer no constraints: γ Peg, V2052 Oph, and the prototype β Cep. These three stars have both the lowest values for the rate of period change but also with the greatest error bars. Similarly, the stars that fall into the third category include those with the highest rates of period change: α Lup, σ Sco, and BW Vul. While the [Jerzykiewicz \(1999\)](#) sample consists of

Table 2. Measured fundamental properties of two β Cephei stars from the literature.

Property	ξ^1 CMa ^a	δ Cet ^b
Mass (M_{\odot})	13–15	7.9
T_{eff} (K)	$25\,900 \pm 100$	$21\,675 \pm 400$
Radius (R_{\odot})	8.7 ± 0.7	–
$\log L/L_{\odot}$	4.46 ± 0.07	3.6

References. ^(a) Shultz et al. (2015); ^(b) Levenhagen et al. (2013).

only eight stars (nine measured rates of period change), there are significant hints that there is something amiss in stellar evolution models that was not found by Eggleton & Percy (1973), even though their models did predict high rates of period change. This is because their models predict that stars evolve as blue supergiant stars within the β Cephei instability strip. All of the β Cephei stars in the sample were classified as either luminosity class III or IV. The three β Cep stars with high rates of period change are luminosity class III but so is the prototype β Cep that has a measured rate of period change that is consistent with zero. There is no obvious connection between rates of period change and evolutionary state of the stars.

Is there anything special about the three β Cephei stars: α Lup, σ Sco, and BW Vul? These three stars appear to pulsate consistently in the fundamental mode, but there is nothing apparently unique about these stars relative to the others in the sample. The mystery deepens when one notes that the other β Cephei stars in our sample tend to have magnetic fields, which are not modeled in this work, and many are slow rotators. The stars that are consistent with none of the stellar evolution models are the ones that are expected to be most easily fit.

The three stars with rates of period change that are consistent with predictions from all stellar evolution models, γ Peg, V2052 Oph, and β Cep, also provide no clear answers. There have been claims that γ Peg has a weak magnetic field (Butkovskaya & Plachinda 2007), but Neiner et al. (2014) reported a null measurement. The star also has a measured rotation rate of about 3 km s^{-1} (Telting et al. 2006; Handler 2009). These observations suggest the stellar evolution models computed in this work should be sufficient for analyzing the stellar properties. On the other hand, V2052 Oph is a magnetic He-strong star (Neiner et al. 2012). Briquet et al. (2012) measured rotation rates of about $70\text{--}75 \text{ km s}^{-1}$ and smaller amounts of convective core overshooting than for other β Cephei variable stars. The prototype β Cep is similarly a magnetic He-strong star with a varying magnetic field of about 100 G (Henrichs et al. 2013). The three β Cep stars that cannot be constrained, i.e., that are consistent with *all* of the computed models, appear to have little in common.

Of the two stars for which we find rates of period change that can measure stellar properties: ξ^1 CMa and δ Cet, the former has a measured magnetic field strength of about 100 G (Hubrig et al. 2006; Silvester et al. 2009) and very slow rotation. This behavior is consistent with spin down due to the magnetic field interactions (Shultz et al. 2015). There have been no measurements of magnetic fields in δ Cet, but it does have a nitrogen excess and slow rotation rate that are consistent with known magnetic β Cep stars (Morel et al. 2006; Levenhagen et al. 2013).

In the next sections, we explore in more detail the predicted stellar properties for ξ^1 CMa and δ Cet in combination with measurements of rotation rates and measurements of fundamental parameters from seismic and other analyses.

4.1. Properties of ξ^1 CMa

Shultz et al. (2015) presented a Baade-Wesselink analysis of ξ^1 CMa using HIPPARCOS photometry and radial velocity measurements (Saesen et al. 2006). They measure a mean effective temperature of $25.9 \pm 0.1 \text{ kK}$, a radius $R = 8.7 \pm 0.7 R_{\odot}$ and a luminosity $\log L/L_{\odot} = 4.46 \pm 0.07$. The authors also measure the age of ξ^1 CMa to be about 12.6 Myr from isochrones (Ekström et al. 2012). Furthermore, they note that the star is rotating slowly with a period of about 60 days .

Using the measured rate of period change and our stellar evolution models assuming zero rotation only we measure an age between $8\text{--}18 \text{ Myr}$, that is consistent with results from the Ekström et al. (2012) stellar isochrones. However, when we include the effective temperature as an additional constraint of the stellar parameters, we measure a luminosity between $\log L/L_{\odot} = 4.40\text{--}4.60$, but with a younger age of $8\text{--}12 \text{ Myr}$. Furthermore, we measure the mass of ξ^1 CMa to be $M = 13\text{--}15 M_{\odot}$ with a low probability of it being more massive if there is no convective core overshooting in the star.

This comparison is striking because of how consistent the measured rate of period change is with predictions from our models while for other cases predicted rates are too low relative to observations. Another issue is that ξ^1 CMa has a strong magnetic field, which is not included in our models. If the magnetic field is strong enough then one might expect it to affect the pulsation properties of the star, but these findings suggest the magnetic field plays no role in the period change. It is not clear why the models provide such consistent predictions of stellar parameters for this star but not for others (Mathis & Neiner 2015).

4.2. Properties of δ Cet

The second case where predicted rates of period change appear to constrain stellar fundamental parameters is for δ Cet. Levenhagen et al. (2013) fit spectroscopic observations with non-LTE models and measured $T_{\text{eff}} = 21\,675 \pm 400 \text{ K}$ and $\log g = 4.03 \pm 0.08$. Other reports measure similar values for the effective temperature but gravities as small as $\log g = 3.7$ (Niemczura & Daszyńska-Daszkiewicz 2005; Morel et al. 2006; Hubrig et al. 2009). Levenhagen et al. (2013) also measured a projected rotation velocity of about $v \sin i = 13 \text{ km s}^{-1}$, suggesting that the star is slowly rotating or is being viewed nearly pole-on. Comparing their results to stellar evolution models, they also infer a mass $= 7.9 M_{\odot}$ and a luminosity of $\log L/L_{\odot} = 3.6$.

When we compare our stellar evolution models to both measurements of period change for δ Cet, we find weakly constrained values of stellar parameters. The predicted age, from $7\text{--}18 \text{ Myr}$, is consistent with previous results (Levenhagen et al. 2013). The effective temperature is predicted to range from $\log T_{\text{eff}} = 4.32\text{--}4.42$, and the luminosity is $\log L/L_{\odot} = 4.2\text{--}5$. The effective temperature is consistent with observations but the luminosity is significantly greater. There is no improvement if we consider only stellar evolution models with no rotation as opposed to the case of ξ^1 CMa.

If we confine our models to only those with effective temperatures that are consistent with the Levenhagen et al. (2013) measurements, we better constrain the other stellar parameters. In this case, we measure the mass of δ Cet to be $M = 10\text{--}12 M_{\odot}$ and its luminosity is $\log L/L_{\odot} = 4.18\text{--}4.25$. The luminosity is still significantly greater than that measured by Levenhagen et al. (2013) using Schaller et al. (1992) stellar evolution tracks. The differences could be due to different prescriptions of interior mixing, opacities and input physics, making it unlikely that our

results indicate some unknown underlying issue with the stellar evolution models.

While a magnetic field has not been directly measured for δ Cet, the star shows nitrogen enhancement which is suggestive of one (Morel et al. 2006; Levenhagen et al. 2013). When coupled with the apparent slow rotation of δ Cet, it appears likely that δ Ceti is similar to ξ^1 CMa. This result further raises questions as to why stellar evolution models appear to fit these two stars, but not the non-magnetic β Cephei variables in the sample.

5. Discussion

The goal of our work was to revisit the results of Eggleton & Percy (1973) comparing stellar evolution models and predicted rates of period change with observations. Eggleton & Percy (1973) found that stellar evolution models fit observed rates of period change for a sample of β Cephei stars well and could explain the significant range of observed rates in terms of both hydrogen-core and hydrogen-shell burning stars. While this evolutionary scenario explains the diversity of period change measurements in β Cephei variable stars, stellar evolution models have advanced significantly in the past forty years.

Since the work of Eggleton & Percy (1973), stellar opacities have changed and new physical processes have been added to stellar evolution models. We computed grids of models at solar composition, for masses from $M = 7\text{--}20 M_{\odot}$. The models included various amounts of convective core overshooting along with different amounts of initial rotation. For every model, we compute relative rates of period change when the stars evolve along the β Cephei instability strip. Both rotation and overshooting extend the main sequence lifetimes of stars, hence act to decrease the relative rates of period change. As such, comparing predicted rates of period change to observations do not allow us to measure the rotation history of a star or the amounts of overshooting because it is not possible to disentangle the stellar mass from the amount of mixing in the stellar evolution models.

We compared our predicted rates to period change measurements for a sample of eight stars (Jerzykiewicz 1999), but found mixed results. For three of the stars: γ Peg, V2052 Oph, and the prototype β Cep, our models could not be applied to constrain any fundamental stellar parameters. The rates of period change for these stars are all consistent with being constant and there are points in the evolution of every star along the β Cephei instability strip where the rate of period change is very low. For three other stars: BW Vul, σ Sco, and α Lup, the predicted rates of period change for all stellar evolution models are too low relative to the measured rates. For two stars ξ^1 CMa and δ Cet, we predicted fundamental stellar parameters by comparing predicted and observed rates of period change. The predicted stellar parameters are consistent with previous independent measurements. We also measure the masses for δ Cet and ξ^1 CMa to be $M = 11 \pm 1 M_{\odot}$ and $M = 14 \pm 1 M_{\odot}$, respectively.

The fits to the measured rates of period change of ξ^1 CMa and δ Cet are both reassuring and surprising. That we find agreement with our models suggests that the analysis is consistent and the models are good representations of real stars. However, there is evidence for strong magnetic fields in the two stars, but these are not taken into account for the stellar evolution models. That the models tend to agree suggests that the presence of a strong magnetic field might not play a significant role in the pulsation of these stars relative to secular stellar evolution.

Those stars with measured rates of period change that are greater than any of those predicted by stellar evolution models

is disconcerting. No amount of rotation or convective core overshooting can resolve that difference nor do these three stars have any common property that could explain the difference. For instance, only V2052 Oph shows evidence for a strong magnetic field. There are a number of reasons as to why theoretical rates of period change might be different from those measured. If these three stars were undergoing mass loss that is about one or two orders of magnitude greater than predicted then the models would have rates of period change similar to that observed. However, there is no reason why mass loss would be enhanced by such an amount for these stars but not the other five, including ξ^1 CMa and δ Cet.

We computed rates of period change by differentiating the period-mean density relation. If β Cephei variable star pulsation followed a different relation then the predicted rates of period change could be significantly different, but again, if we use a different relation then the agreement found for ξ^1 CMa and δ Cet would be lost.

A third option is that the measured rates of period change for these stars not only represent secular period change, but include the effects of possible period drift where the pulsation of the star changes abruptly over short timescales due to some non-evolutionary cause. Odell (2012) analyzed timing measurements for BW Vul and argued that the pulsation period is mostly constant with respect to time, but underwent random fluctuations during various times in the past. These changes are not secular, i.e., not a function of evolution, but this cause is unclear. A similar period drift has been suspected for the classical Cepheid Polaris (Turner et al. 2005; Neilson et al. 2012a). If this period drift is occurring in β Cephei variable stars then it could explain why there are such high rates of period change for only three stars. It could also explain the two β Cephei variable stars for which the rates of period change are well-fit, the stars have not undergone period drifts over the time scales of the observations.

Our results suggest the evolution of β Cephei variable stars and their pulsation properties are more complex than expected, raising questions about their rates of period change. However, our results are based on measurements for only eight stars, necessitating period change measurements for more β Cephei variable stars along with continued observations of the eight stars discussed in Jerzykiewicz (1999). Similarly, our models assume that the period-mean density relation is constant and is a valid measure of secular period change that should be verified by direct modeling of pulsation and evolution.

References

- Aerts, C., Waelkens, C., & de Pauw, M. 1994, *A&A*, 286, 136
Aerts, C., Thoul, A., Daszyńska, J., et al. 2003, *Science*, 300, 1926
Aerts, C., Marchenko, S. V., Matthews, J. M., et al. 2006, *ApJ*, 642, 470
Briquet, M., Morel, T., Thoul, A., et al. 2007, *MNRAS*, 381, 1482
Briquet, M., Neiner, C., Aerts, C., et al. 2012, *MNRAS*, 427, 483
Brott, I., de Mink, S. E., Cantiello, M., et al. 2011a, *A&A*, 530, A115
Brott, I., Evans, C. J., Hunter, I., et al. 2011b, *A&A*, 530, A116
Butkovskaya, V. V., & Plachinda, S. I. 2007, *A&A*, 469, 1069
Cassisi, S., & Salaris, M. 2011, *ApJ*, 728, L43
Cox, A. N., Morgan, S. M., Rogers, F. J., & Iglesias, C. A. 1992, *ApJ*, 393, 272
Demarque, P., & Percy, J. R. 1965, *AJ*, 70, 136
Dziembowski, W. A., Moskalik, P., & Pamyatnykh, A. A. 1993, *MNRAS*, 265, 588
Eddington, A. S. 1918, *MNRAS*, 79, 2
Eggleton, P. P., & Percy, J. R. 1973, *MNRAS*, 161, 421
Ekström, S., Georgy, C., Eggenberger, P., et al. 2012, *A&A*, 537, A146
Fossati, L., Castro, N., Morel, T., et al. 2015, *A&A*, 574, A20
Frischknecht, U., Hirschi, R., Meynet, G., et al. 2010, *A&A*, 522, A39
Grevesse, N., & Sauval, A. J. 1998, *Space Sci. Rev.*, 85, 161
Handler, G. 2009, *MNRAS*, 398, 1339

- Handler, G., Shobbrook, R. R., Uytterhoeven, K., et al. 2012, *MNRAS*, **424**, 2380
- Heger, A., & Langer, N. 2000, *ApJ*, **544**, 1016
- Henrichs, H. F., de Jong, J. A., Verdugo, E., et al. 2013, *A&A*, **555**, A46
- Huang, R. Q., & Weigert, A. 1983, *A&A*, **127**, 309
- Hubrig, S., Briquet, M., Schöller, M., et al. 2006, *MNRAS*, **369**, L61
- Hubrig, S., Briquet, M., De Cat, P., et al. 2009, *Astron. Nachr.*, **330**, 317
- Hubrig, S., Ilyin, I., Schöller, M., et al. 2011, *ApJ*, **726**, L5
- Huenemoerder, D. P., Oskinova, L. M., Ignace, R., et al. 2012, *ApJ*, **756**, L34
- Hunter, I., Brott, I., Lennon, D. J., et al. 2008, *ApJ*, **676**, L29
- Jerzykiewicz, M. 1999, *New Astron. Rev.*, **43**, 455
- Jerzykiewicz, M., & Pigulski, A. 1999, *New Astron. Rev.*, **43**, 461
- Langer, N. 2012, *ARA&A*, **50**, 107
- Levenhagen, R. S., Künzel, R., & Leister, N. V. 2013, *New Astron.*, **18**, 55
- Lovekin, C. C., & Goupil, M.-J. 2010, *A&A*, **515**, A58
- Martins, F., Schaerer, D., Hillier, D. J., et al. 2005, *A&A*, **441**, 735
- Mathis, S., & Neiner, C. 2015, in *IAU Symp.*, **307**, 420
- Mazumdar, A., Briquet, M., Desmet, M., & Aerts, C. 2006, *A&A*, **459**, 589
- Miglio, A., Montalbán, J., Eggenberger, P., & Noels, A. 2009, *Comm. Asteroseismol.*, **158**, 233
- Morel, T., Butler, K., Aerts, C., Neiner, C., & Briquet, M. 2006, *A&A*, **457**, 651
- Moskalik, P., & Dziembowski, W. A. 1992, *A&A*, **256**, L5
- Neilson, H. R., & Langer, N. 2012, *A&A*, **537**, A26
- Neilson, H. R., Cantiello, M., & Langer, N. 2011, *A&A*, **529**, L9
- Neilson, H. R., Engle, S. G., Guinan, E., et al. 2012a, *ApJ*, **745**, L32
- Neilson, H. R., Langer, N., Engle, S. G., Guinan, E., & Izzard, R. 2012b, *ApJ*, **760**, L18
- Neiner, C., Alecian, E., Briquet, M., et al. 2012, *A&A*, **537**, A148
- Neiner, C., Monin, D., Leroy, B., Mathis, S., & Bohlender, D. 2014, *A&A*, **562**, A59
- Niemczura, E., & Daszyńska-Daszkiewicz, J. 2005, *A&A*, **433**, 659
- Odell, A. P. 2012, *A&A*, **544**, A28
- Oskinova, L. M., Nazé, Y., Todt, H., et al. 2014, *Nat. Commun.*, **5**, 4024
- Pamyatnykh, A. A. 2007, *Comm. Asteroseismol.*, **150**, 207
- Pigulski, A. 1992, *A&A*, **261**, 203
- Pigulski, A. 1993, *A&A*, **274**, 269
- Pigulski, A. 1994, *A&A*, **292**, 183
- Prada Moroni, P. G., Gennaro, M., Bono, G., et al. 2012, *ApJ*, **749**, 108
- Saesen, S., Briquet, M., & Aerts, C. 2006, *Comm. Asteroseismol.*, **147**, 109
- Schaller, G., Schaerer, D., Meynet, G., & Maeder, A. 1992, *A&AS*, **96**, 269
- Shultz, M., Wade, G., Rivinius, T., et al. 2015, in *IAU Symp.*, **307**, 399
- Silvester, J., Neiner, C., Henrichs, H. F., et al. 2009, *MNRAS*, **398**, 1505
- Stothers, R. 1965, *ApJ*, **141**, 671
- Suárez, J. C., Moya, A., Amado, P. J., et al. 2009, *ApJ*, **690**, 1401
- Telting, J. H., Schrijvers, C., Ilyin, I. V., et al. 2006, *A&A*, **452**, 945
- Townsend, R. H. D., Owocki, S. P., & Howarth, I. D. 2004, *MNRAS*, **350**, 189
- Turner, D. G., Savoy, J., Derrah, J., Abdel-Sabour Abdel-Latif, M., & Berdnikov, L. N. 2005, *PASP*, **117**, 207
- Vink, J. S., de Koter, A., & Lamers, H. J. G. L. M. 2000, *A&A*, **362**, 295
- Yoon, S.-C., & Langer, N. 2005, *A&A*, **443**, 643