The radial gradient of the near-surface shear layer of the Sun

A. Barekat1, J. Schou1, and L. Gizon1,2

1 Max-Planck-Institut für Sonnensystemforschung, Justus-von-Liebig-Weg 3, 37077 Göttingen, Germany
e-mail: barekat@mps.mpg.de
2 Institut für Astrophysik, Georg-August-Universität Göttingen, 37077 Göttingen, Germany

Received 20 August 2014 / Accepted 2 October 2014

ABSTRACT

Context. Helioseismology has provided unprecedented information about the internal rotation of the Sun. One of the important achievements was the discovery of two radial shear layers: one near the bottom of the convection zone (the tachocline) and one near the surface. These shear layers may be important ingredients for explaining the magnetic cycle of the Sun.

Aims. We measure the logarithmic radial gradient of the rotation rate ($\text{dln}\Omega / \text{dln} r$) near the surface of the Sun using 15 years of $f$ mode rotational frequency splittings from the Michelson Doppler Imager (MDI) and four years of data from the Helioseismic and Magnetic Imager (HMI).

Methods. We model the angular velocity of the Sun in the upper ~10 Mm as changing linearly with depth and use a multiplicative optimally localized averaging inversion to infer the gradient of the rotation rate as a function of latitude.

Results. Both the MDI and HMI data show that $\text{dln}\Omega / \text{dln} r$ is close to $-1$ from the equator to 60° latitude and stays negative up to 75° latitude. However, the value of the gradient is different for MDI and HMI for latitudes above 60°. Additionally, there is a significant difference between the value of $\text{dln}\Omega / \text{dln} r$ using an older and recently reprocessed MDI data for latitudes above 30°.

Conclusions. We could reliably infer the value of $\text{dln}\Omega / \text{dln} r$ up to 60°, but not above this latitude, which will hopefully constrain theories of the near-surface shear layer and dynamo. Furthermore, the recently reprocessed MDI splitting data are more reliable than the older versions which contained clear systematic errors in the high degree $f$ modes.

Key words. Sun: helioseismology – Sun: interior – Sun: rotation

1. Introduction

Helioseismology has had a significant impact on our understanding of the internal structure and dynamics of the Sun. One of the most important results has been the inference of the rotation profile (Schou et al. 1998). Two shear layers have been identified, one located near the base of the convection zone (the tachocline) and one near the surface. These shear layers may be important ingredients for explaining the magnetic cycle of the Sun.

Aims. We measure the logarithmic radial gradient of the rotation rate ($\text{dln}\Omega / \text{dln} r$) near the surface of the Sun using 15 years of $f$ mode rotational frequency splittings from the Michelson Doppler Imager (MDI) and four years of data from the Helioseismic and Magnetic Imager (HMI).

Methods. We model the angular velocity of the Sun in the upper ~10 Mm as changing linearly with depth and use a multiplicative optimally localized averaging inversion to infer the gradient of the rotation rate as a function of latitude.

Results. Both the MDI and HMI data show that $\text{dln}\Omega / \text{dln} r$ is close to $-1$ from the equator to 60° latitude and stays negative up to 75° latitude. However, the value of the gradient is different for MDI and HMI for latitudes above 60°. Additionally, there is a significant difference between the value of $\text{dln}\Omega / \text{dln} r$ using an older and recently reprocessed MDI data for latitudes above 30°.

Conclusions. We could reliably infer the value of $\text{dln}\Omega / \text{dln} r$ up to 60°, but not above this latitude, which will hopefully constrain theories of the near-surface shear layer and dynamo. Furthermore, the recently reprocessed MDI splitting data are more reliable than the older versions which contained clear systematic errors in the high degree $f$ modes.
results obtained by CT, we also use older version of the MDI data. The differences between these versions come from various improvements to the analysis, as described in Larson & Schou (2009) and (Larson & Schou, in prep.). We refer to the older version as “old MDI” and to the latest “new MDI”.

The $f$ modes we use cover the range $117 \leq l \leq 300$ for MDI and $123 \leq l \leq 300$ for HMI. We note that the number of available modes changes with time because of noise.

3. Analysis of $f$ mode data

The odd $a$-coefficients are related to the angular velocity $\Omega$ by

$$2\pi a_{l,2s+1} = \int_{-1}^{1} \int_{l}^{r} K_{l,s}(r,u)\Omega(r,u)udr,$$

where the kernels $K_{l,s}$ are known functions, $u = \cos \theta$, $\theta$ is the co-latitude, and $r$ is the distance to the center of the Sun divided by the photospheric radius. Using the results of Pijpers (1997), one can show that the kernels can be separated in the variables $r$ and $u$, $K_{l,s}(r,u) = F_{l,s}(r)G_{l,s}(u)$, (3) where the functions $F_{l,s}$ and $G_{l,s}$ are the radial and latitudinal parts of the kernels. The function $F_{l,s}$ is

$$F_{l,s}(r) = \left[ F_{l,1}(r) - F_{l,2}(r)(2s+2)(2s+1)/2 \right] P_{l,2s+1},$$

where $F_{l,1}$, $F_{l,2} v_{l,2s+1}$ are given by

$$F_{l,1}(r) = \rho(r)^2 \left[ \xi_1^2(r) - 2\xi_1(r)\eta_1(r)/L + \eta_1^2(r) \right]/l_I,$$

$$F_{l,2}(r) = \rho(r)^2 \eta_1^2(r)/(L/l_I),$$

$$v_{l,2s+1} = \frac{(\xi_1^2)(2l+1)!/(2s+2)!/(l-s+1)!}{\xi_1^2 + \eta_1^2} \cdot \frac{1}{s!(s+1)!} \cdot \frac{\pi}{2} \cdot \frac{1}{l_I}.$$

In the above equations $\rho$ is the density, $L = \sqrt{l(l+1)}$, $\xi$ and $\eta$ are the radial and horizontal displacement eigenfunctions as defined by Pijpers (1997), and $l_I = \int_{-1}^{1} \rho(r)^2 \left[ \xi_1^2(r) + \eta_1^2(r) \right] dr$. The latitudinal part of the kernels is given by

$$G_{l,s}(u) = \frac{(4s+3)}{2(2s+2)(2s+1)} \frac{1}{(1-u)^{l/2}} P_{l,2s+1}^s(u),$$

where $P_{l,2s+1}^s$ are associated Legendre polynomials of degree $2s+1$ and order one. As seen later, the form of Eq. (3) is useful in that the latitudinal part of the kernels is independent of $l$.

We use $f$ modes to calculate $\ln \Omega/\ln r$ close to the surface of the Sun in several steps. In the first step, we assume that the rotation rate changes linearly with depth at each latitude

$$\Omega(r,u) = \Omega_0(u) + (1-r)\Omega_{1}(u),$$

where $\Omega_0$ is the slope and $\Omega_0$ is the value of the rotation rate at the surface. Combining Eq. (9) with Eqs. (2) and (3) we obtain

$$\bar{\Omega}_{ls} \equiv \frac{2\pi a_{l,2s+1}}{\beta_{ls}} = \langle \Omega_{0} \rangle_s + (1-r)\Omega_{1}(u)s,$$

where $\beta_{ls} = \int_{0}^{r} F_{l,s}(r)dr$ and $\bar{\Omega}_{ls} = \beta_{ls}^{-1} \int_{0}^{r} F_{l,s}(r)dr$ is the center of gravity of $F_{l,s}$. The quantities $\Omega_{0}(u)$ and $\Omega_{1}(u)$ are the latitudinal averages

$$\langle \Omega_{0} \rangle_s = \int_{-1}^{1} G_{l,s}(u)\Omega_{0}(u)du,$$

$$\langle \Omega_{1} \rangle_s = \int_{-1}^{1} G_{l,s}(u)\Omega_{1}(u)du.$$
five common 72 day periods between 2010 April 30 and 2011 April 24. The results are consistent up to \( \sim 60^\circ \) within 2\( \sigma \), but show significant inconsistencies at higher latitudes. An analysis using only the common modes and the average errors does not significantly reduce this high latitude discrepancy. This indicates that there are systematic errors in at least one of the data sets, as opposed to only differences in the mode coverage or error estimates. The source of the systematic errors is unknown, but could be related to inaccurate estimates of the optical distortion of the instruments or similar geometric errors (Larson & Schou, in prep.). Another possible source is the different duty cycles. For example, the last three data sets for MDI had duty cycles of 88%, 73%, and 81%, while the corresponding HMI duty cycles were 97%, 99%, and 96%. In either case we conclude that the results above \( \sim 60^\circ \) should be treated with caution.

The results presented here are significantly different from those obtained by CT. They found that \( \text{dln} \Omega / \text{dln} r \) is close to \(-1\) from the equator to \(30^\circ\) latitude, while our result shows this up to \(60^\circ\) latitude. They also found that their results changed significantly if they restricted the degree range. To investigate the origin of these differences we examine the effects of each of the differences between their data and analysis and ours.

First, we compare the results of applying our method and theirs to the 23 time periods they used (covering the period 1996 May 1 to 2001 April 4). Corbard & Thompson (2002) first made an error weighted time average of an older version of the MDI data and then applied their Eq. (9). If we repeat this procedure on the same data sets we obtain results visually identical to theirs. The difference between the data sets used by CT and old MDI is that a few modes were accidentally removed from the older set. We then changed the processing order to first apply their Eq. (9) to old MDI and then make an unweighted time average. As shown in Fig. 4, this results in minor differences at high latitude and an analysis applying each change separately shows that only the change from weighted fits to unweighted fits leads to a noticable difference.

![Fig. 2. Time average of \( \text{dln} \Omega / \text{dln} r \) versus target latitude, obtained from 15 years (1996–2011) of MDI data (black dots) and 4 years (2010–2014) of HMI data (red dots). The error bars are 1\( \sigma \).](image)

![Fig. 3. Comparison of \( \text{dln} \Omega / \text{dln} r \) versus target latitude for MDI (black dots) and HMI (red dots) from the five common 72 day time series (indicated by the nominal beginning dates). Error bars are 1\( \sigma \).](image)

![Fig. 4. Estimates of \( \text{dln} \Omega / \text{dln} r \) versus target latitude obtained from 23 MDI data sets using various methods. Blue diamonds show the values measured from Fig. 4 of CT, while black pluses show the results of changing the data sets and averaging, as described in the text. Green squares and dark blue stars show the results of our analysis of the old MDI data for the full and restricted modes, respectively. Filled and open red circles show the corresponding results for the new MDI data.](image)
As almost all the differences between the results obtained by CT and ours come from the differences between old and new MDI, we compare the α-coefficients directly. As an example, Fig. 5 shows α₃ for the modes with 150 ≤ l ≤ 300 for all 74 periods. The main differences between new and old MDI appear for l > 270. In the new MDI data most of the missing modes (shown in black) in the old MDI data are recovered and the yearly oscillatory pattern disappears. These differences clearly show that the old MDI data have significant systematic errors in the high degree f modes. We also note that the new values of α₃ are shifted towards higher values.

5. Conclusion

We analyze 15 years (1996–2011) of reprocessed MDI data and 4 years (2010–2014) of HMI data to infer the logarithmic radial gradient of the angular velocity of the Sun in the upper ~10 Mm. By using data from two instruments and applying a different method than CT did, we confirm their value of dlnΩ/dln r ~ −1 at low latitudes (<30°); unlike CT, we show that dlnΩ/dln r stays nearly constant and close to −1 up to 60° latitude.

With further analysis we conclude that the inconsistency between their results and ours for latitudes above 30° is due to systematic errors in the old MDI data. This implies that work done using old MDI data should be revisited. By comparing the results obtained from new MDI and HMI data, we also conclude that at least one of the data sets is likely still suffering from some systematic errors which leads to the discrepancy above 60° latitude.

The measured value dlnΩ/dln r ~ −1 is inconsistent with the standard picture of angular momentum conservation where dlnΩ/dln r is ~2 (Foukal 1977; Gilman & Foukal 1979). More recently, hydrodynamical mean-field simulations of a larger part of the convection zone by Kitchatinov & Rüdiger (2005) show a NSSL with a negative radial gradient of the angular velocity from the equator to 80° latitude. Their theory (Kitchatinov & Rüdiger 1993, 1999; Kitchatinov 2013) states that the formation of the NSSL is due to the balance of the A-effect (Ruediger 1989) and the eddy viscosity. However, producing a NSSL with the correct radial gradient remains a challenge for direct numerical simulations of the Sun (e.g., Warnecke et al. 2013; Guerrero et al. 2013) and we still do not understand why the value of dlnΩ/dln r at the surface is nearly constant and so close to −1.

We note here that we measure dlnΩ/dln r only in the upper ~10 Mm which is only about one third of the NSSL. To extend this range one would need to use p modes, which unfortunately have much more noise. A preliminary analysis shows that dlnΩ/dln r shows little solar cycle variation, though there are weak hints of a torsional oscillation-like signal. However, this requires further analysis.

Acknowledgements. We thank T. P. Larson for discussions regarding details of the MDI and HMI peakbagging, T. Corbard and M. J. Thompson for clarifying details of their analysis, and A. Birch for various discussions. SOHO is a project of international cooperation between ESA and NASA. The HMI data are courtesy of NASA/SDO and the HMI science team.

References

Christensen-Dalsgaard, J., & Schou, J. 1988, in Seismology of the Sun and Sun-Like Stars, ed. E. J. Rolfe, ESA SP, 286, 149
Corbard, T., & Thompson, M. J. 2002, Sol. Phys., 205, 211
Ruediger, G. 1989, Differential rotation and stellar convection. Sun and the solar stars (Berlin: Akademie Verlag)