Binary black holes in nuclei of extragalactic radio sources

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ABSTRACT

If we assume that nuclei of extragalactic radio sources contain binary black hole systems, the two black holes can eject VLBI components, in which case two families of different VLBI trajectories will be observed. Another important consequence of a binary black hole system is that the VLBI core is associated with one black hole, and if a VLBI component is ejected by the second black hole, one expects to be able to detect the offset of the origin of the VLBI component ejected by the black hole that is not associated with the VLBI core. The ejection of VLBI components is perturbed by the precession of the accretion disk and the motion of the black holes around the center of gravity of the binary black hole system. We modeled the ejection of the component taking into account the two perturbations and present a method to fit the coordinates of a VLBI component and to deduce the characteristics of the binary black hole system. Specifically, this is the ratio $T_p/T_b$ where $T_p$ is the precession period of the accretion disk and $T_b$ is the orbital period of the binary black hole system, the mass ratio $M_1/M_2$, and the radius of the binary black hole system $R_{\text{bin}}$. From the variations of the coordinates as a function of time of the ejected VLBI component, we estimated the inclination angle $i_\gamma$ and the bulk Lorentz factor $\gamma$ of the modeled component. We applied the method to component S1 of 1823+568 and to component C5 of 3C 279, which presents a large offset of the space origin from the VLBI core. We found that 1823+568 contains a binary black hole system whose size is $R_{\text{bin}} \approx 60 \mu$as ($\mu$as is a microarcsecond) and 3C 279 contains a binary black hole system whose size is $R_{\text{bin}} \approx 420 \mu$as.

\textbf{Key words.} astrometry – galaxies: jets – galaxies: individual: 1823+568 – galaxies: individual: 3C 279

1. Introduction

VLBI observations of compact radio sources show that the ejection of VLBI components does not follow a straight line, but undulates. These observations suggest a precession of the accretion disk. To explain the precession of the accretion disk, we assumed that the nuclei of radio sources contain binary black hole systems (BBH system, see Fig. 1).

A BBH system produces three perturbations of the VLBI ejection due to:

1. the precession of the accretion disk;
2. the motion of the two black holes around the center of gravity of the BBH system; and
3. the motion of the BBH system in the galaxy.

In this article, we do not take into account the possible third perturbation due to the motion of the BBH system in the galaxy.

A BBH system induces several consequences, which are that

1. even if the angle between the accretion disk and the plane of rotation of the BBH system is zero, the ejection does not follow a straight line (due to the rotation of the black holes around the center of gravity of the BBH system);
2. the two black holes can have accretion disks with different angles with the plane of rotation of the BBH system and can eject VLBI components; in that case we observe two different families of trajectories; a good example of a source with two families of trajectories is 3C 273, whose components C5 and C9 follow two different types of trajectories (see Fig. 2); and
3. if the VLBI core is associated with one black hole, and if the VLBI component is ejected by the second black hole, there will be an offset between the VLBI core and the origin of the ejection of the VLBI component; this offset will correspond to the radius of the BBH system.

The precession of the accretion disk can be explained using a single rotating black hole (Lense-Thirring effect) or by the magnetically driven precession (Caproni et al. 2006). However, a single black hole and a BBH system have completely different consequences. In the case of a BBH system, one has an extra perturbation of the ejected component due to the motions of the black holes around the center of gravity of the BBH system. One can expect to observe two different families of trajectories (if the two
black holes eject VLBI components) and an offset of the origin of the ejected component if it is ejected by the black hole that is not associated with the VLBI core.

We modeled the ejection of the VLBI component using a geometrical model that takes into account the two main perturbations due to the BBH system, i.e.

1. the precession of the accretion disk; and
2. the motion of the two black holes around the center of gravity of the BBH system.

Modeling the ejection of VLBI components using a BBH system has been developed in previous articles, for instance Britzen et al. (2001) modeled 0420-014, Lobanov & Roland (2005) modeled 3C 345, and Roland et al. (2008) modeled 1803+784. Observationnal VLBI studies have been performed to directly detect BBH systems in active galactic nuclei (Burke-Spolaor 2011; Tingay & Wayth 2011).

In Sect. 2 we recall the main lines of the model. The details of the model can be found in Roland et al. (2008).

We determined the free parameters of the model by comparing the observed coordinates of the VLBI component with the calculated coordinates of the model.

This method requires knowing of the variations of the two coordinates of the VLBI component as a function of time. Because these observations contain the kinematical information, we will be able to estimate the inclination angle of the source and the bulk Lorentz factor of the ejected component.

In this article we present a method to solve this problem, either for a precession model or for a BBH system model, based on understanding the space of the solutions.

Practically, two different cases can occur when we try to solve this problem.

1. Either the VLBI component is ejected from the VLBI core, or the offset is smaller than or on the order of the smallest error bars of the VLBI positions of the ejected component (case I);
2. or the VLBI component is ejected with an offset larger than the smallest error bars of the VLBI positions of the ejected component (case II).

Case II is much more complicated to solve than case I, because the observed coordinates contain an unknown offset that is larger than the error bars. Therefore, we first have to find the offset, then correct the VLBI data from the offset, and finally find the solution corresponding to the corrected data.

We present the method for solving the problem in Sect. 3. To illustrate case I, we solve the fit of component S1 of 1823+568 using MOJAVE data in Sect. 4. To illustrate case II, we solve the fit of component C5 of 3C 279 using MOJAVE data in Sect. 5.

2. Model

2.1. Introduction: two-fluid model

We describe the ejection of a VLBI component in the framework of the two-fluid model (Sol et al. 1989; Pelletier & Roland 1989, 1990; Pelletier & Sol 1992). The two-fluid description of the outflow is adopted with the following assumptions:

1. The outflow consists of an $e^- - e^+$ plasma (hereafter the beam) moving at a highly relativistic speed (with corresponding Lorentz factor $\gamma_b \lesssim 30$) surrounded by an $e^- - p$ plasma (hereafter the jet) moving at a mildly relativistic speed of $v_j \lesssim 0.4 \times c$.
2. The magnetic field lines are parallel to the flow in the beam and the mixing layer, and are toroidal in the jet (see Fig. 3).

Muxlow et al. (1988) and Roland et al. (1988) found that the Cygnus A hot spots could be explained by a an $e^- - p$ plasma moving at a mildly relativistic speed, i.e. $v_j \lesssim 0.4 \times c$. Consequently, the two-fluid model was introduced to explain superluminal radio sources observed in the nuclei of radio sources.

The $e^- - p$ jet carries most of the mass and the kinetic energy ejected by the nucleus. It is responsible for the formation of kpc-jets, hot spots, and extended lobes (Roland & Hetem 1996). The relativistic $e^+$ beam moves in a channel through the mildly relativistic jet and is responsible for the formation of superluminal sources and their $\gamma$-ray emission (Roland et al. 1994). The relativistic beam can propagate when the magnetic field $B$ is parallel to the flow in the beam and in the mixing layer between the beam and the jet, and when it is greater than a critical value (Pelletier et al. 1988; Achatz & Schlickeiser 1993). The magnetic field in the jet becomes rapidly toroidal as a function of distance from the core (Pelletier & Roland 1990).

\[ \text{The bulk Lorentz factor is limited to 30 to ensure the propagation stability of the relativistic beam in the subrelativistic jet.} \]
The observational evidence for the two-fluid model has been discussed by e.g. Roland & Hetem (1996). Observational evidence for relativistic ejection of an electron beam comes from the γ-ray observations of MeV sources (Roland & Hermsen 1995; Skibo et al. 1997) and from VLBI polarization observations (Attridge et al. 1999).

The possible existence of VLBI components with two different apparent speeds has been investigated by Marcowith et al. (1995, 1998) for Centaurus A. The formation of X-ray and γ-ray spectra, assuming relativistic ejection of an electron beam, has been investigated by Marcowith et al. (1995, 1998).

2.2. Geometry of the model

We call Ω the angle between the accretion disk and the orbital plane (XOY) of the BH system. The component is ejected on a cone (the precession cone) with its axis in the Z ′ OZ plane and of opening angle Ω. We assumed that the line of sight is in the plane (YOZ) and forms an angle η with the axis Z ′ OZ (see Fig. 4). The axis η corresponds to the mean ejection direction of the VLBI component projected in a plane perpendicular to the line of sight, so the plane perpendicular to the line of sight is the plane (ηOX). We call ΔΞ the rotation angle in the plane perpendicular to the line of sight to transform the coordinates η and X into coordinates N (north) and W (west), which are directly comparable with the VLBI observations. We have

\[ W = -x \cos(\Delta \Xi) + (\zeta \sin(i_\eta) + \gamma \cos(i_\eta)) \sin(\Delta \Xi), \]

\[ N = x \sin(\Delta \Xi) + (\zeta \sin(i_\eta) + \gamma \cos(i_\eta)) \cos(\Delta \Xi). \]

The sign of the coordinate W was changed from Roland et al. (2008) to use the same definition as VLBI observations.

2.3. General perturbation of the VLBI ejection

For VLBI observations, the origin of the coordinates is black hole 1, i.e. the black hole ejecting the VLBI components. For the sake of simplicity, we assumed that the two black holes have circular orbits, i.e. ε = 0. Therefore, the coordinates of the moving components in the frame of reference where black hole 1 is considered the origin are (Roland et al. 2008)

\[ x_c = [R_\odot(z_c) \cos(\omega_0 t - k_\odot z(t) + \phi_0) + x_1 \cos(\omega_0 t - k_\odot z(t) + \phi_0) - x_1 \cos(\phi_0)] \exp(-t/T_\odot), \]

\[ y_c = [R_\odot(z_c) \sin(\omega_0 t - k_\odot z(t) + \phi_0) + y_1 \sin(\omega_0 t - k_\odot z(t) + \phi_0) - y_1 \sin(\phi_0)] \exp(-t/T_\odot), \]

\[ z_c = z_\odot(t), \]

where

- \( R_\odot(z) \) is the amplitude of the precession perturbation, given by \( R_\odot(z) = R_\odot(\zeta(t))/\cos(z_\odot(t)), \) with \( R_\odot = R_\odot(2 \tan \Omega); \)
- \( \omega_0 \) is \( \omega_0 = 2\pi/T_\odot, \) where \( T_\odot \) is the precession period, and \( k_\odot \) is defined by \( k_\odot = 2\pi/T_\odot V_\odot, \) where \( V_\odot \) is the speed of the propagation of the perturbations;
- \( \omega_1 \) is \( \omega_1 = 2\pi/T_1, \) where \( T_1 \) is the BBH system period and \( k_1 \) is defined by \( k_1 = 2\pi/T_1 V_1; \)
- \( T_\odot \) is the characteristic time of the damping of the perturbation;
- \( x_1 \) and \( y_1 \) are given by

\[ x_1 = y_1 = -\frac{M_1}{M_1 + M_2} \times \frac{T_\odot^2}{4\pi^2} G(M_1 + M_2) \frac{1/3}{D_\odot}. \]

We define with \( R_{\text{bin}} \) the distance between the two black holes as the size of the BBH system. It is

\[ R_{\text{bin}} = \left[ \frac{T_\odot^2}{4\pi^2} G(M_1 + M_2) \frac{1/3}{D_\odot} \right]. \]

In mas units (milli arc second units), it is

\[ R_{\text{bin}} \approx 2.06 \times 10^8 \left[ \frac{T_\odot^2}{4\pi^2} G(M_1 + M_2) \frac{1/3}{D_\odot} \right] /D_\odot. \]

where \( D_\odot = D_i / (1 + \zeta^2) \) is the angular distance, \( D_i \) is the luminosity distance, and \( \zeta \) is the redshift of the source. The differential equation governing the evolution of \( \zeta_\odot(t) \) can be obtained by defining the speed of the component, namely

\[ v_\zeta(t) = \left( \frac{dx_\zeta(t)}{dt} \right)^2 + \left( \frac{dy_\zeta(t)}{dt} \right)^2 + \left( \frac{dz_\zeta(t)}{dt} \right)^2, \]

where \( v_\zeta \) is related to the bulk Lorentz factor by \( v_\zeta/c = \sqrt{(1-1/y_\zeta)}. \)

Using (3)–(5), we find from (9) that \( dz_\zeta/dt \) is the solution of the equation

\[ A \left( \frac{dz_\zeta}{dt} \right)^2 + B \left( \frac{dz_\zeta}{dt} \right) + C = 0. \]

The calculation of the coefficients A, B and C can be found in Appendix A of Roland et al. (2008).

Equation (10) admits two solutions corresponding to the jet and the counter-jet. Following Camenzind & Krokenberger (1992), if we call θ the angle between the velocity of the component and the line of sight, we have

\[ \cos(\theta(t)) = \left( \frac{dy_c}{dt} \sin i_\eta + \frac{dz_c}{dt} \cos i_\eta \right) / v_c. \]
The Doppler beaming factor $\delta_c$, characterizing the anisotropic emission of the moving component, is

$$\delta_c(t) = \frac{1}{\gamma_c \left[ 1 - \beta_c \cos(\theta(t)) \right]}, \quad (12)$$

where $\beta_c = v_c/c$. The observed flux density is

$$S_c = \frac{D_l}{D_i} \delta_c(t)^2 \alpha_c \left( 1 + z \right)^{-\alpha}, \quad (13)$$

where $D_l$ is the luminosity distance of the source, $z$ its redshift, $\alpha_c$ is the emissivity of the component, and $\alpha_t$ is the synchrotron spectral index (it is related to the flux density by $S \propto \nu^{-\alpha_t}$). As the component is moving relativistically toward the observer, the observed time is shortened and is given by

$$t_{\text{obs}} = \int_0^t \left[ 1 - \beta_c \cos(\theta(t')) \right] \left( 1 + z \right) \, dt'. \quad (14)$$

2.4. Coordinates of the VLBI component

Solving (10), we determine the coordinate $\hat{z}_c(t)$ of a point-source component ejected relativistically in the perturbed beam. Then, using (3) and (4), we can find the coordinates $x_c(t)$ and $y_c(t)$ of the component. In addition, for each point of the trajectory, we can calculate the derivatives $dx_c/\,dt$, $dy_c/\,dt$, $dz_c/\,dt$ and then deduce $\cos \theta$ from (11), $\delta_c$ from (12), $S_c$ from (13) and $t_{\text{obs}}$ from (14).

After calculating the coordinates $x_c(t)$, $y_c(t)$ and $z_c(t)$, they can be transformed to $x_c(t)$ (west) and $n_c(t)$ (north) coordinates using (1) and (2).

As explained in Britzen et al. (2001), Lobanov & Roland (2005), and Roland et al. (2008), the radio VLBI component has to be described as an extended component along the beam. We call $n_{\text{rad}}$ the number of points (or integration steps along the beam) for which we integrate to model the component. The coordinates $W_c(t), N_c(t)$ of the VLBI component are then

$$W_c(t) = \sum_{i=1}^{n_{\text{rad}}} w_c(t_i) / n_{\text{rad}} \quad (15)$$

and

$$N_c(t) = \sum_{i=1}^{n_{\text{rad}}} n_c(t_i) / n_{\text{rad}} \quad (16)$$

and can be compared with the observed coordinates of the VLBI component, which correspond to the radio peak intensity coordinates provided by model-fitting during the VLBI data reduction process.

When, in addition to the radio, optical observations are available that peak in the light curve, this optical emission can be modeled as the synchrotron emission of a point source ejected in the perturbed beam, see Britzen et al. (2001) and Lobanov & Roland (2005). This short burst of very energetic relativistic $e^+$ is followed immediately by a very long burst of less energetic relativistic $e^+$. This long burst is modeled as an extended structure along the beam and is responsible for the VLBI radio emission. In that case the origin $t_0$ of the VLBI component is the beginning of the first peak of the optical light curve and is not a free parameter of the model.

2.5. Parameters of the model

In this section, we list the possible free parameters of the model. They are

- $i_o$, the inclination angle;
- $\phi_o$, the phase of the precession at $t = 0$;
- $\Delta \phi$ the rotation angle in the plane perpendicular to the line of sight (see Eqs. (1) and (2));
- $\Omega$ the opening angle of the precession cone;
- $R_s$ the maximum amplitude of the perturbation;
- $T_p$ the precession period of the accretion disk;
- $T_d$ the characteristic time for the damping of the beam perturbation;
- $M_1$ the mass of the black hole ejecting the radio jet;
- $M_2$ the mass of the secondary black hole;
- $\gamma_c$ the bulk Lorentz factor of the VLBI component;
- $\psi_o$ the phase of the BBH system at $t = 0$;
- $t_o$ the time of the origin of the ejection of the VLBI component;
- $V_o$ the propagation speed of the perturbations;
- $n_{\text{rad}}$ the number of steps to describe the extension of the VLBI component along the beam;
- $\Delta W$ and $\Delta N$ the possible offsets of the origin of the VLBI component.

We will see that the parameter $V_o$ can be used to study the degeneracy of the solutions, so we can keep it constant to find the solution. The range of values that we study for parameter $V_o$ is $0.10 \times c \leq V_o \leq 0.45 \times c^2$.

The parameter $n_{\text{rad}}$ is known when the size of the VLBI component is known.

This means that, practically, the problem we have to solve is a 15 free parameter problem.

We have to investigate the different possible scenarios with regard to the sense of the rotation of the accretion disk and the sense of the orbital rotation of the BBH system. These possibilities correspond to $\pm \omega_p(t - z/V_o)$ and $\pm \omega_b(t - z/V_o)$. Because the sense of the precession is always opposite to the sense of the orbital motion (Katz 1997), we study the two cases denoted by $++$ and $+-$, where we have $\omega_p(t - z/V_o), -\omega_b(t - z/V_o)$ and $-\omega_p(t - z/V_o), \omega_b(t - z/V_o)$, respectively.

3. Method for solving the problem

3.1. Introduction

In this section, we explain the method for fitting VLBI observations using either a precession model or a BBH system model. The software is freely available on request to J. Roland\(^3\).

This method is a practical one that provides solutions, but the method is not unique and does not guarantee that all possible solutions are found.

We calculate the projected trajectory on the plane of the sky of an ejected component and determine the parameters of the model to simultaneously produce the best fit with the observed west and north coordinates. The parameters found minimize

$$\chi^2 = \chi^2(W_c(t)) + \chi^2(N_c(t)), \quad (17)$$

where $\chi^2(W_c(t))$ and $\chi^2(N_c(t))$ are the $\chi^2$ calculated by comparing the VLBI observations with the calculated coordinates $W_c(t)$ and $N_c(t)$.

\(^3\) We limit ourselves to nonrelativistic hydrodynamics in this model.

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and $N_c(t)$ of the component. For instance, to find the inclination angle that provides the best fit, we minimize $\chi^2(i_o)$. A good determination of the $1\sigma$ (standard deviation) error bar can be obtained using the definition

$$
(\Delta i_o)_{1\sigma} = |i_o(x_{\text{max}}^2 + 1) - i_o(x_{\text{min}}^2)|,
$$

which provides two values $(\Delta i_o)_{1\sigma+}$ and $(\Delta i_o)_{1\sigma-}$ (see Lampton et al. 1976 and Hébrard et al. 2002).

The concave parts of the surface $\chi^2(i_o)$ contain a minimum. We can find solutions without a minimum; they correspond to the convex parts of the surface $\chi^2(i_o)$ and are called mirage solutions.

To illustrate the properties of the surface $\chi^2(i_o)$ we plot in Fig. 5 a possible example of a profile of the solution $\chi^2(i_o)$. In Fig. 5, there are two possible solutions for which $\chi^2(Sol 1) = \chi^2(Sol 2)$, solution 2 is more robust than solution 1, i.e. it is the deepest one, and it will be the solution we adopt.

![Fig. 5. Example of a possible profile of the solution $\chi^2(i_o)$. There are two possible solutions for which $\chi^2(Sol 1) = \chi^2(Sol 2)$. They correspond to the concave parts of the surface $\chi^2(i_o)$. However, solution 2 is more robust than solution 1, i.e. it is the deepest one, and it will be the solution we adopt.](image)

Generally, for any value of the parameters, the surface $\chi^2(\Delta)$ is convex and does not present a minimum. Moreover, when we are on the convex part of the surface $\chi^2(\Delta)$, one of the important parameters of the problem can diverge. The two important parameters of the problem that can diverge are

1. the bulk Lorentz factor of the $e^\pm$ beam, which has to be $\gamma_b \leq 30$. This limit is imposed by the stability criterion for the propagation of the relativistic beam in the subrelativistic $e^- - p$ jet;
2. the total mass of the BBH system.

The most frequent case of divergence we can find corresponds to $\gamma_b \to \infty$. These mirage solutions are catastrophic and must be rejected. As we will see, generally, we have to study the robustness of the solution in relation to the parameters $T_p/T_b, M_1/M_2, \gamma$ and $i_o$.

3.2. Solution of the precession model

In a first step, we fit a simple precession model without a BBH system. This corresponds to the precession induced by a spinning BH (Lense-Thirring effect) or by the magnetically driven precession (Caproni et al. 2006). This has the advantage of determining whether the solution corresponds to case I or to case II and of preliminarily determining the inclination angle and the bulk Lorentz factor of the ejected component.

We have to investigate the different possible scenarios with regard to the sense of the rotation of the accretion disk. These possibilities correspond to $\pm \omega_p(t - z/V_a)$. Accordingly, we study the two cases.

Assuming a simple precession model, these are the steps to fit the coordinates $X(t)$ and $Y(t)$ of a VLBI component:

1. Determining the solution $\chi^2(i_o)$ and the time origin of the component ejection. In this section we assume that $V_a = 0.1c$ (as $\chi^2(V_a)$ remains constant when $V_a$ varies, any value of $V_a$ can be used, see details in the next paragraph). We calculate $\chi^2(i_o)$, i.e., we minimize $\chi^2(i_o)$ when the inclination angle varies gradually between two values. At each step of $i_o$, we determine each free parameter $A$ such that $\partial \chi^2/\partial A = 0$. Firstly, the important parameter to determine is the time origin of the ejection of the VLBI component. We compare the times of the observed peak flux with the modeled peak flux. The time origin is obtained when the two peak fluxes occur at the same time. The solutions corresponding to case II show a significant difference between the time origin of the ejection of the VLBI component deduced from the fit of the peak flux and the time origin obtained from the interpolation of the core separation. Second, we can make a first determination of the inclination angle and of the bulk Lorentz factor.

2. Determining the family of solutions. The solution previously found is not unique and shows a degeneracy. The parameter $V_a$ can be used to study the degeneracy of the solution. Indeed, if we calculate $\chi^2(V_a)$ when $V_a$ varies, we find that $\chi^2(V_a)$ remains constant. For the inclination angle found in the previous section and the parameters of the corresponding solution, we calculate $\chi^2(V_a)$ when $V_a$ varies between $0.01c \leq V_a \leq 0.45c$ and deduce the range of the precession period.

3. Determining the possible offset of the origin of the VLBI component. In this section, we keep $V_a = 0.1c$ and using the inclination angle previously found and the corresponding solution, we calculate $\chi^2(\Delta x, \Delta y)$ when $\Delta x$ and $\Delta y$...
3.3. Solution of the BBH model

We have to investigate the different possible scenarios with regard to the sense of the rotation of the accretion disk and the sense of the orbital rotation of the BBH system. Because the sense of the precession is always opposite to the sense of the orbital motion, we study the two cases where we have $\omega_b(t-z/V_o)$, $-\omega_b(t-z/V_o)$, and $\omega_p(t-z/V_o)$, respectively.

Assuming a BBH model, this is the method for fitting the coordinates $X(t)$ and $Y(t)$ of a VLBI component:

1. **Determining the BBH system parameters for various values of $T_p/T_b$.** In this section, we keep the inclination angle previously found and $V_a = 0.1c$. We determine the BBH system parameters for different values of $T_p/T_b$, namely $T_p/T_b = 1.01, 2.2, 4.6, 10, 22, 46, 100, and 220$ for a BBH system with $M_1 = M_2$ (these values of $T_p/T_b$ are chosen because they are equally spaced on a logarithmic scale). Generally, the BBH systems obtained with a low value of $T_p/T_b$, namely $T_p/T_b = 1.01, 2.2, or 4.6$ are systems with a large radius and the BBH systems obtained with a high value of $T_p/T_b$, namely $T_p/T_b = 10, 22, 46, 100, or 220$ are systems with a small radius.

2. **Determining the possible solutions:** the $\chi^2(T_p/T_b)$ diagram. In this section, we keep the inclination angle previously found, $V_a = 0.1c$ and $M_1 = M_2$. The crucial parameter for finding the possible solutions is $T_p/T_b$, i.e., the ratio of the precession period and the binary period. Starting from the solutions found in the previous section, we calculate $\chi^2(T_p/T_b)$ when $T_p/T_b$ varies between 1 and 300. We find that the possible solutions characterized by a specific value of the ratio $T_p/T_b$. We note that some of the solutions can be mirage solutions, which have to be detected and excluded.

3. **Determining the possible offset of the space origin.** In this section, we keep the inclination angle previously found, $V_a = 0.1c$ and $M_1 = M_2$. Starting with the solution found in the previous section, we calculate $\chi^2(\Delta x, \Delta y)$ when $\Delta x$ and $\Delta y$ vary ($\Delta x$ and $\Delta y$ are the possible offsets of the VLBI origin). If we find that an offset of the origin is needed, we correct the BBH coordinates by the offset to continue. Note that determining the offsets of the VLBI coordinates does not depend on the value of the inclination angle.

4. **Determining the range of possible values of $T_p/T_b$.** In this section, we keep $V_a = 0.1c$, $M_1 = M_2$. Previously, we found a solution characterized by a value of $T_p/T_b$ for a given inclination. Therefore we calculate $\chi^2(i_o)$ when $i_o$ varies with a variable ratio $T_p/T_b$. We obtain the range of possible values of $T_p/T_b$ and the range of possible values of $i_o$.

5. **Preliminary determination of $i_o$.** $T_p/T_b$ and $M_1/M_2$. In this section, we keep $V_a = 0.1c$. This section is the most complicated one and differs for solutions corresponding to case I and case II. We indicate the main method and the main results (the details are provided in Sect. A.7 for the fit of component S1 of 1823+568 solutions and in Sect. B.7 for the fit of component C5 of 3C 279). We calculate $\chi^2(i_o)$ for various values of $T_p/T_b$ and $M_1/M_2$. Generally, we find that there exist critical values of the parameters $T_p/T_b$ and $M_1/M_2$, which separate the domains for which the solutions exist or become mirage solutions. The curves $\chi^2(i_o)$ show a minimum for given values $(i_o)_{\text{min}}$ and if necessary, we study the robustness of the solution in relation to the parameter $\gamma$, therefore we calculate $\chi^2(\gamma)$ at $i_o = (i_o)_{\text{min}}$ for the corresponding values of $T_p/T_b$ and $M_1/M_2$. When these critical values are obtained, we find the domains of $T_p/T_b$ and $M_1/M_2$, which produce the solutions whose robustness is greater than $1.7\sigma$ and the corresponding inclination angle $i_o$.

6. **Determining a possible new offset correction.** Using the solution found in the previous section, we calculate again $\chi^2(\Delta x, \Delta y)$ when $\Delta x$ and $\Delta y$ vary. When a new offset of the origin is needed, we correct the VLBI coordinates by the new offset to continue. Note that this new offset correction is smaller than the first one found previously.

7. **Characteristics of the final solution to the fit of the VLBI component.** We are now able to find the BBH system parameters that produce the best solution for the fit with the same method as described in point 5, Preliminary determination of $i_o$. $T_p/T_b$ and $M_1/M_2$.

8. **Determining the family of solutions.** The solution previously found is not unique and shows a degeneracy. The parameter $V_a$ can be used to study the degeneracy of the solution. Indeed, when we calculate $\chi^2(V_a)$ for varying $V_a$, we find that $\chi^2(V_a)$ remains constant. Using the solution found in the previous section and the parameters of the corresponding solution, we calculate $\chi^2(V_a)$ when $V_a$ varies between $0.01c \leq V_a \leq 0.45c$ and deduce the range of the precession period, the binary period, and the total mass of the BBH system.

9. **Determining the size of the accretion disk.** Because we know the parameters of the BBH system, we can deduce the rotation period of the accretion disk and its size.

4. **Method – Case I**

4.1. **Introduction:** fitting the component S1 of 1823+568

Case I corresponds to a VLBI component ejected either from the VLBI core or to one where the offset of the origin of the ejection is smaller than or on the order of the smallest error bars of the VLBI component coordinates. It is the simplest case to solve. To illustrate the method of solving the problem corresponding to case I, we fit the component S1 of the source 1823+568 (Figs. 6 and 7).

4.2. **VLBI data of 1823+568**

1823+568 is an quasar at a redshift of $0.664 \pm 0.001$ (Lawrence et al. 1986). The host galaxy is elliptical according to HST observations (Falomo et al. 1997). The jet morphology on kpc-scales is complex – a mirrored S in observations with the MTRLI at 1666 MHz and with the VLA at 2 and 6 cm (O’Dea et al. 1988). The largest extension of 1823+568 is $15''$, corresponding to 93 kpc. On pc-scales the jet is elongated and points in a southern direction from the core (Pearson & Readhead 1988) – in accordance with the kpc-structure. Several components could be identified in the jet, e.g., Gabuzda et al. (1989) and Gabuzda et al. (1994), Gabuzda & Cawthorne (1996), Forstad et al. (2005). A VSOP Space VLBI image of 1823+568 has been obtained by Lister et al. (2009a). All identified components show strong polarization. The linear polarization is parallel to the jet edge direction. Most of the components show slow apparent superluminal motion. The fast component S1 moved with an apparent velocity of about 20 ± 2c until 2005 and subsequently decreases (Glück 2010). Twenty-two VLBA observations obtained
at 15 GHz within the 2-cm MOJAVE survey between 1994.67 and 2010.12 have been re-analyzed and model-fitted to determine the kinematics of the individual components. For details of the data reduction and analysis see Glück (2010).

The radio map of 1823+568, observed 9 May 2003, is shown in Fig. 6. The data are taken from Glück (2010).

### 4.3. Preliminary remarks

The redshift of the source is \( z_s \approx 0.664 \), and using for the Hubble constant \( H_0 \approx 72 \, \text{km/s/Mpc} \), the luminosity distance of the source is \( D_L \approx 3882 \, \text{Mpc} \) and the angular distance is \( D_A = D_L /(1+z)^2 \).

For details of the values of the data and of their error bars see Glück (2010). At 15 GHz, calling the beam size \( Beam \), we adopted for the minimum values \( \Delta_{\text{min}} \) of the error bars of the observed VLBI coordinates, the values in the range:

\[
\text{Beam}/15 \leq \Delta_{\text{min}} \leq \text{Beam}/12,
\]

see Sect. C for details concerning this choice.

For 1823+568, observations were performed at 15 GHz and the beam size is mostly circular and equal to \( Beam \approx 0.5 \, \text{mas} \). We adopted as minimum values of the error bars the values \( \Delta W_{\text{min}} = \text{Beam}/12 \approx 40 \, \mu\text{as} \) and \( \Delta N_{\text{min}} = \text{Beam}/12 \approx 40 \, \mu\text{as} \) for the west and north coordinates of component S1, i.e., when the error bars obtained from the VLBI data reduction were smaller than \( \Delta W_{\text{min}} \) or \( \Delta N_{\text{min}} \), they were enlarged to the minimum values. The minimum values were chosen empirically, but the adopted values were justified a posteriori by comparing the \( \chi^2 \) value of the final solution and the number of constraints to make the fit and to obtain a reduced \( \chi^2 \) close to 1. For the component S1, we have \( \chi^2_{\text{final}} \approx 51 \) for 56 constraints, the reduced \( \chi^2 \) is \( \chi^2 = \chi^2_{\text{final}}/56 \approx 0.91 \). Lister & Homan (2005) suggested that the positional error bars should be about 1/5 of the beam size. However, if we had chosen \( \Delta W_{\text{min}} = \Delta N_{\text{min}} = \text{Beam}/5 \approx 100 \, \mu\text{as} \), we would have

\[
(\chi^2)_{\text{final}} \ll 56 \text{, indicating that the minimum error bars would be overestimated (see details in Sect. C).}
\]

To obtain a constant projected trajectory of the VLBI component in the plane perpendicular to the line of sight, the integration step to solve Eq. (10) changes when the inclination angle varies. The integration step was \( \Delta t = 0.8 \, \text{yr} \) when \( i_0 = 5^\circ \). When \( i_0 \) varied, it was \( \Delta t = 0.8(\sin(5^\circ)/\sin(i_0)) \) yr.

The trajectory of component S1 is not long enough to constrain the parameter \( T_d \), i.e., the characteristic time for the damping of the beam perturbation. We fit assuming that \( T_d \leq 2500 \, \text{yr} \); this value produced a good trajectory shape.

The time origin of the ejection of the component S1, deduced from the interpolation of VLBI data, is \( t_0 \approx 1995.6 \) (Fig. 7).

Close to the core, the size of S1 is \( \approx 0.24 \, \text{mas} \), therefore we assumed that \( n_{\text{rad}} = 75 \), where \( n_{\text{rad}} \) is the number of steps to describe the extension of the VLBI component along the beam. At \( i_0 = 5^\circ \) with an integration step \( \Delta t = 0.8 \, \text{yr} \), we calculated the length of the trajectory corresponding to each integration step. The size of the component is the sum of the first \( n_{\text{rad}} = 75 \) lengths.

### 4.4. Final fit of component S1 of 1823+568

Here we present the solution to the fit of S1, the details for the fit can be found in Sect. A.

We studied the two cases \( \pm \alpha_p (t - z/V_a) \). The final solution of the fit of component S1 using a BBH system corresponds to \( \pm \alpha_p (t - z/V_a) \) and \( -\alpha_p (t - z/V_a) \).

The main characteristics of the solution of the BBH system associated with 1823+568 are that

- the radius of the BBH system is \( R_{\text{bin}} \approx 60 \, \mu\text{as} \approx 0.42 \, \text{pc} \);
- the VLBI component S1 is not ejected by the VLBI core, and the offsets of the observed coordinates are \( \Delta W \approx +5 \, \mu\text{as} \) and \( \Delta N \approx 60 \, \mu\text{as} \);
- the ratio \( T_p/T_b \) is \( 8.88 \leq T_p/T_b \leq 9.88 \); and
- the ratio \( M_1/M_2 \) is \( 0.905 \leq M_1/M_2 \leq 0.25 \).

The results of the fits obtained for \( T_p/T_b = 8.88 \) and \( T_p/T_b = 9.88 \) are given in Sect. A.9. The solutions found with \( T_p/T_b \approx 8.88 \) are slightly more robust, but both solutions can be used.
To continue, we arbitrarily adopted the solution with \( T_p/T_b \approx 8.88 \) and \( M_1/M_2 \approx 0.17 \). We deduced the main parameters of the model, which are that

- the inclination angle is \( i_o \approx 3.98^\circ \);
- the angle between the accretion disk and the rotation plane of the BBH system is \( \Omega \approx 0.28^\circ \) (this is also the opening angle of the precession cone);
- the bulk Lorentz factor of the VLBI component is \( \gamma_c \approx 17.7 \); and
- the origin of the ejection of the VLBI component is \( t_o \approx 1995.7 \).

The variations of the apparent speed of component S1 are shown in Fig. 8.

We can determine the Doppler factor (Eq. (12)), and consequently, we can estimate the observed flux density (Eq. (13)). This was used to fit the temporal position of the peak flux and to determine the temporal origin of the ejection of the VLBI component (see Sect. A.1 for the details).

The fit of the two coordinates \( W(t) \) and \( N(t) \) of the component S1 of 1823+568 is shown in Fig. 9. The points are the observed coordinates of component S1 that were corrected by the offsets \( \Delta W \approx +5 \mu as \) and \( \Delta N \approx 60 \mu as \), and the red lines are the coordinates of the component trajectory calculated using the BBH model assuming the solution parameters, i.e., \( T_p/T_b \approx 8.88, M_1/M_2 \approx 0.17, i_o \approx 3.98^\circ \), etc.

Finally, we compared this solution with the solution obtained using the precession model. The \( \chi^2 \) of this fit is about 51 for the fit using the BBH system and about 67 for the precession model (see Sect. A.1), i.e., the BBH system solution is a 4\( \sigma \) better solution.

To fit the ejection of component S1 we used 56 observations (the west and north coordinates corresponding to the 28 epochs of observation), so the reduced \( \chi^2 \) is \( \chi^2_\text{red} = 51/56 \approx 0.91 \), indicating that the minimum values used for the error bars are correct.

### 4.5. Determining the family of solutions

For the inclination angle previously found, i.e., \( i_o \approx 3.98^\circ \), \( T_p/T_b \approx 8.88, M_1/M_2 \approx 0.17 \), and \( R_\text{bin} \approx 60 \mu as \), we gradually varied \( V_o \) between 0.01c and 0.45c. The function \( \chi^2(V_o) \) remained constant, indicating a degeneracy of the solution. We deduced the range of variation of the BBH system parameters. They are given in Table 1.

The period of the BBH system is not obviously related to a possible periodicity of the radio or the optical light curve.

#### 4.6. Determining the size of the accretion disk

From the knowledge of the mass ratio \( M_1/M_2 \approx 0.17 \) and the ratio \( T_p/T_b \approx 8.88 \), we calculated in the previous section the mass of the ejecting black hole \( M_1 \), the orbital period \( T_b \), and the precession period \( T_p \) for each value of \( V_o \).

We calculated the rotation period of the accretion disk, \( T_{\text{disk}} \), is given by (Britzen et al. 2001)

\[
T_{\text{disk}} \approx \frac{4}{3} \frac{M_1 + M_2}{M_2} T_b \frac{T_p}{T^2_{\text{p}}}.
\]  

(20)

Thus we calculated the rotation period of the accretion disk, and assuming that the mass of the accretion disk is \( M_{\text{disk}} \ll M_1 \), the size of the accretion disk \( R_{\text{disk}} \) is

\[
R_{\text{disk}} \approx \left( \frac{T^2_{\text{disk}}}{4\pi^2} GM_1 \right)^{1/3}.
\]  

(21)

We found that the size of the accretion disk does not depend on \( V_o \) and is \( R_{\text{disk}} \approx 0.090 \) pc \( \approx 0.013 \) mas.
5. The method – Case II

5.1. Introduction: application to component C5 of 3C 279

Case II corresponds to an ejection of the VLBI component with an offset of the origin of the component larger than the smallest error bars of the VLBI component coordinates. This is the most difficult case to solve because data have to be corrected by an unknown offset. That is larger than the smallest error bars.

When we apply the precession model, there are two signatures of case II, which are

1. the problem of the time origin of the VLBI component, and
2. the shape of the curve \( \chi^2(i_o) \).

Using the precession model, we modeled the flux and compared the time position of the first peak flux with the time position of the observed peak flux. If the origin time deduced from interpolating the VLBI data was very different than the origin time deduced from the precession model, we concluded that there is a time origin problem (see Sect. B.1). We show that this origin-time problem is related to the offset of the space origin of the VLBI component, i.e., the VLBI component is not ejected by the VLBI core and this offset is larger than the smallest error bars (see Sect. B.3).

When the offset of the space origin is larger than the smallest error bars of the component positions and the VLBI coordinates are not corrected by this offset, the curve \( \chi^2(i_o) \) can have a very characteristic shape:

1. the inclination angle is limited to a specific interval, i.e., \( i_{\text{min}} \leq i_o \leq i_{\text{max}} \).
2. when \( i_o \rightarrow i_{\text{max}} \) and when \( i_o \rightarrow i_{\text{min}} \), the bulk Lorentz factor of the VLBI component diverges, i.e., \( \gamma_i \rightarrow \infty \); and
3. the function \( \chi^2(i_o) \) does not have a minimum in the interval \( i_{\text{min}} \leq i_o \leq i_{\text{max}} \).

See Fig. B.1 corresponding to the precession model applied to component C5 of 3C 279.

5.2. MOJAVE data of 3C 279

The radio quasar 3C 279 (\( z = 0.536 \) Marziani et al. 1996) is one of the brightest extragalactic radio sources and has been observed and studied in detail for decades. Superluminal motion in the outflow of the quasar was found by Whitney et al. (1971) and Cohen et al. (1971). Thanks to the increasing resolution and sensitivity of modern observation techniques, a more complex picture of 3C 279 appeared, including multiple superluminal features moving along different trajectories downstream the jet (Unwin et al. 1989). The apparent speed of these components span an interval between 4c and 16c (Cotton et al. 1979; Wehrle et al. 2001).

We used the MOJAVE observations of 3C 279 (Lister et al. 2001). Seventy-six VLBA observations obtained at 15 GHz within the 2-cm MOJAVE survey between 1999.25 and 2007.64 were re-analyzed and model-fitted to determine the coordinates of the VLBI components. We used the NRAO Astronomical Image Processing System (AIPS) to calibrate the data. We performed an amplitude calibration and applied a correction for the atmospheric opacity for the high-frequency data (\( \nu > 15 \) GHz). The parallactic angle correction was taken into account before we calibrated the phases using the pulse-scale signal and a final fringe fit. The time- and frequency-averaged data were imported to DIFMAP (Shepherd 1997), were used the CLEAN and MODELFIT algorithm for imaging and model fitting, respectively.

The fully calibrated visibilities were fitted in DIFMAP using the algorithm MODELFIT and 2D circular Gaussian components. These components were characterized by their flux density, \( S_{\text{mod}} \), position \( r_{\text{mod}} \), position angle (PA), \( \theta_{\text{mod}} \) (measured from north through east), and their full-width at half-maximum (FWHM). Since the number of fitted Gaussians was initially not limited, we only then added a new component when the \( \chi^2 \) value decreased significantly. This approach led to a minimum number of Gaussians that can be regarded as a reliable representation of the source. We modeled each epoch separately to avoid aliasing effects. The kinematics of the source could thus be analyzed by tracking the fitted components. The average beam for the 15 GHz observations is 0.51 mas x 1.34 mas.

The radio map of 3C 349, observed 15 June 2003, is shown in Fig. 10. The data are taken from Lister et al. (2009a).

5.3. Preliminary remarks

The redshift of 3C 279 is \( z \approx 0.536 \), and using for the Hubble constant \( H_0 \approx 72 \) km/s/Mpc, the luminosity distance of the source is \( D_L \approx 3070 \) Mpc and the angular distance is \( D_A = D_L/(1 + z)^2 \).

\[
T_p(V_o) \approx 540 \text{,000 yr} \quad \approx 6700 \text{ yr} \\
T_b(V_o) \approx 60 \text{,600 yr} \quad \approx 750 \text{ yr} \\
(M_1 + M_2)(V_o) \approx 1.6 \times 10^9 M_\odot \quad \approx 1.05 \times 10^9 M_\odot
\]
For details of the values of the data see Lister et al. (2009a). Because the observations were performed at 15 GHz and the beam size was 0.51 mas×1.34 mas, we adopted for the minimum values of the error bars the values \(\Delta W_{\text{min}} \approx \text{Beam}/15 \approx 34 \mu\text{as}\) and \((\Delta N)_{\text{min}} \approx \text{Beam}/15 \approx 89 \mu\text{as}\) for the west and north coordinates of component C5. The adopted values were justified a posteriori by comparing the \(\chi^2\) value of the final solution and the number of constraints to make the fit and to obtain a reduced \(\chi^2\) close to 1. For the component C5, we have \((\chi^2)_{\text{final}} \approx 150\) for 152 constraints, thus the reduced \(\chi^2\) is: \((\chi^2)_{\text{r}} \approx 0.99\). It has been suggested by Lister & Homan (2005) that the positional error should be within 20% of the convolving beam size, i.e., \(\approx \text{Beam}/5\). See Sect. C for details concerning the choice adopted in this article and the determination of the \(\chi^2\), the characteristics of the solution using minimum error bars are as large as \(\approx \text{Beam}/5\).

The integration step used to solve Eq. (10) is \(\Delta t = 0.8\) yr when \(i_o = 5^\circ\). When \(i_o\) varies, it is \(\Delta t = 0.8(\sin(5^\circ)/\sin(i_o))\) yr.

The trajectory of component C5 is not long enough to constrain the parameter \(T_d\), i.e., the characteristic time for the damping of the beam perturbation. We fit assuming that \(T_d \leq 2000\) yr.

The time origin of the ejection of the component C5 cannot be deduced easily from the interpolation of VLBI data (Lister et al. 2009b). However, we show in Sect. B.1 how, using the precession model, it is possible to obtain the minimum time origin of the VLBI component by comparing the time position of the calculated first peak flux with the observed time position of the first peak flux.

Close to the core, the size of C5 is \(\approx 0.25\) mas, therefore we assumed that \(n_{\text{rad}} = 75\), where \(n_{\text{rad}}\) is the number of steps to describe the extension of the VLBI component along the beam.

### 5.4. Final fit of component C5 of 3C 279

Here we present the solution to the fit of C5, the details for the fit can be found in Sect. B. The fit of component C5 using a BBH system corresponds to \(-\omega_p(t - z/V_o)\) and \(+\omega_p(t - z/V_o)\).

The main characteristics of the solution of the BBH system associated with 3C 279 are that

- the radius of the BBH system is \(R_{\text{bin}} \approx 420 \mu\text{as} \approx 2.7\) pc;
- the VLBI component C5 is not ejected by the VLBI core and the offsets of the observed coordinates are \(\Delta W \approx \pm 405 \mu\text{as}\) and \(\Delta N \approx \pm 110 \mu\text{as}\);
- the ratio \(T_p/T_b\) is \(T_p/T_b \approx 140\); and
- the ratio \(M_1/M_2\) is \(M_1/M_2 \approx 2.75\).

The results of the fits obtained for \(T_p/T_b \approx 140\) and \(M_1/M_2 \approx 2.75\) are given in Appendix B.9.

Adopting the solution with \(T_p/T_b \approx 140\) and \(M_1/M_2 \approx 2.75\), we deduced the main parameters of the model.

- The inclination angle is \(i_o \approx 10.4^\circ\).
- The angle between the accretion disk and the rotation plane of the BBH system is \(\Omega \approx 2.4^\circ\) (this is also the opening angle of the precession cone).
- The bulk Lorentz factor of the VLBI component is \(\gamma_c \approx 16.7\).
- The origin of the ejection of the VLBI component is \(t_o \approx 1999.0\).

The variations of the apparent speed of component C5 are shown in Fig. 12.

We can determine the Doppler factor (Eq. (12)), and consequently we can estimate the observed flux density (Eq. (13)). Using the precession model, we fitted the temporal position of the peak flux and determined the temporal origin of the ejection of the VLBI component (see Sect. B.1 for the details). Using the BBH model, we calculated and plotted in Fig. 13 the flux variations of C5 using Eq. (A.1). We found that the time origin of the ejection of component C5 is \(t_o \approx 1999.03\). Although Eq. (A.1) is a rough estimate of the flux density variations, it allows us

- to check the time origin of the ejection of the VLBI component found using the BBH model;
- to compare the time position of the modeled first peak flux with the observed first peak flux;
- to obtain a good shape of the variation of the flux density during the first few years and explain the difference between the radio and the optical light curves. In some cases, in addition to the radio, optical observations show a light curve with peaks separated by about one year, see for instance the cases of 0420-016 (Britzen et al. 2001) and 3C 345 (Lobanov & Roland 2005). Using Eq. (A.1), the optical emission can be modeled as the synchrotron emission of a point source ejected in the perturbed beam (Britzen et al. 2001; Lobanov & Roland 2005). This short burst of very energetic relativistic \(e^+\) is followed immediately by a very long burst of less...
energetic relativistic e\(^+\). This long burst is modeled as an extended structure along the beam and is responsible for the VLBI radio emission.

The fit of both coordinates \(W(t)\) and \(N(t)\) of component C5 of 3C 279 are shown in Fig. 14. The points are the observed coordinates of component C5 that were corrected for the offsets \(\Delta W \approx +405\ \mu\text{as}\) and \(\Delta N \approx +110\ \mu\text{as}\), the red lines are the coordinates of the component trajectory calculated using the BBH model assuming the solution parameters, i.e., \(T_p/T_b \approx 140\), \(M_1/M_2 \approx 2.75\), \(i_0 \approx 10.4^\circ\), etc.

Finally, we compared this solution with the solution obtained using the precession model. The \(\chi^2_{\text{min}}(\Delta a)\) is about 151.4 for the fit using the BBH system and \(>1000\) for the precession model (see Sect. B.1). To fit the ejection of component C5 we used 152 observations (76 epochs), so the reduced \(\chi^2\) is \(\chi^2/\text{degrees of freedom} = 152 \approx 0.996\).

5.4.1. Determining the family of solutions

The solution is not unique, but there exists a family of solutions. For the inclination angle previously found, i.e., \(i_0 \approx 10.4^\circ\) and using the parameters of the corresponding solution, i.e., \(T_p/T_b \approx 140\), \(M_1/M_2 \approx 2.75\) and \(R_{\text{bin}} \approx 420\ \mu\text{as}\), we gradually varied \(V_a\) between 0.01\(c\) and 0.45\(c\). The function \(\chi^2(V_a)\) remains constant, indicating a degeneracy of the solution, and we deduced the range of variation of the BBH system parameters. They are given in Table 2.

5.4.2. Determining the size of the accretion disk

From the knowledge of the mass ratio \(M_1/M_2 \approx 2.75\) and the ratio \(T_p/T_b \approx 140\), we calculated in the previous section the mass of the ejecting black hole \(M_1\), the orbital period \(T_b\), and the precession period \(T_p\) for each value of \(V_a\).

We calculated the rotation period of the accretion disk, \(T_{\text{disk}}\), using (20). Assuming that the mass of the accretion disk is

\[ M_{\text{disk}} \ll M_1, \text{ the size of the accretion disk } R_{\text{disk}} \text{ is calculated using (21).} \]

We found that the size of the accretion disk does not depend on \(V_a\) and is \(R_{\text{disk}} \approx 0.26\ \text{pc} \approx 0.041\ \text{mas}\).

5.4.3. Comparing of the trajectories of C5 and C10

We see from Fig. 11 that

- components C5 and C6 probably follow probably the same trajectories;
- component C10 follows a different trajectory than C5 and C6.

Thus, using the MOJAVE data (Lister et al. 2009b), we plot in Fig. 15 the trajectories of C5 and C10. We found that

- component C10 is probably ejected by the VLBI core;
- component C5 is ejected with a large offset from the VLBI core; and
- components C5 and C10 follow two different trajectories and are not ejected from the same origins, indicating that the nucleus of 3C 279 contains a BBH system.

6. Discussion and conclusion

We showed how from the knowledge of the coordinates \(\text{West}(t)\) and \(\text{North}(t)\) of the ejected VLBI component one can find the
characteristics of the BBH system in both cases. To illustrate case I, we fitted component S1 of 1823+568, and to illustrate case II, we fitted component C5 of 3C 279.

From the fit of the coordinates of component S1 of 1823+568, the main characteristics of the final solution of the BBH system associated with 1823+568 are that

- the radius of the BBH system is $R_{\text{BBH}} \approx 60 \mu as \approx 0.42$ pc;
- the VLBI component S1 is not ejected by the VLBI core, and the offsets of the observed coordinates are $\Delta W \approx +5 \mu as$ and $\Delta N \approx 60 \mu as$;
- the ratio $T_p/T_b$ is 8.88 $\leq T_p/T_b \leq 9.88$;
- the ratio $M_1/M_2$ is $0.095 \leq M_1/M_2 \leq 0.25$;
- the inclination angle is $i_0 \approx 40.0^\circ$;
- the bulk Lorentz factor of the VLBI component is $\gamma_c \approx 17.7$; and
- the origin of the ejection of the VLBI component is $t_o \approx 1995.7$.

From the fit of the coordinates of component C5 of 3C 279, the main characteristics of the final solution of the BBH system associated with 3C 279 are that

- the radius of the BBH system is $R_{\text{BBH}} \approx 420 \mu as \approx 2.7$ pc;
- the VLBI component C5 is not ejected by the VLBI core and the offsets of the observed coordinates are $\Delta W \approx +405 \mu as$ and $\Delta N \approx +110 \mu as$;
- the ratio $T_p/T_b$ is $T_p/T_b \approx 140$;
- the ratio $M_1/M_2$ is $M_1/M_2 \approx 2.75$;
- the inclination angle is $i_0 \approx 10.4^\circ$;
- the bulk Lorentz factor of the VLBI component is $\gamma_c \approx 16.7$; and
- the origin of the ejection of the VLBI component is $t_o \approx 1999.0$.

If, in addition to the radio observations, one can obtain optical, X-ray, or $\gamma$-ray observations that show a light curve with peaks, the simultaneous fit of the VLBI coordinates and this light curve put stronger constraints on the characteristics of the BBH system. The high-frequency emission can be modeled as the synchrotron emission or the inverse Compton emission of a point source ejected in the perturbed beam, see Britzen et al. (2001) for PKS 0420-014 and Lobanov & Roland (2005) for 3C 345. This short burst of very energetic relativistic $e^\pm$ is followed immediately by a very long burst of less energetic relativistic $e^\pm$. This long burst is modeled as an extended structure along the beam and is responsible for the VLBI radio emission. The simultaneous fit of the VLBI coordinates and the optical light curve using the same method as the one developed in this article has to be achieved.

Observations of compact radio sources in the first mas show that the VLBI ejections do not follow a straight line, and modeling the ejection shows in each case studied that the nucleus contains a BBH system. Accordingly, Britzen et al. (2001) assumed that all radio sources contain a BBH system. If extragalactic radio sources are associated with galaxies formed after the merging of galaxies and if the formation of extragalactic radio sources is related to the presence of binary black hole systems in their nuclei, we can explain

- why extragalactic radio sources are associated with elliptical galaxies;
- why more than 90% of the quasars are radio-quiet quasars, e.g., Kellermann et al. (1989) and Miller et al. (1990).

Radio-quiet quasars are active nuclei that contain a single black hole and can be associated with spiral galaxies (Peacock et al. 1986). Although it has not been proven yet that radio-quiet quasars only contain a single black hole, the hypothesis for distinguishing between radio-loud and radio-quiet quasars on the basis of the binarity of the central engine is supported by comparing the optical properties of the two classes (Goldschmidt et al. 1999). Recent observations of the central parts of radio galaxies and radio-quiet galaxies show a systematic difference between the two classes (Kharb et al. 2012).

Because Gaia will provide positions of extragalactic radio sources within $\approx 25 \mu as$, the link between the Gaia reference frame from optical observations of extragalactic radio sources and the reference frame obtained from VLBI observations will have to take into account the complex structure of the nuclei of extragalactic radio sources, because with a resolution of $\approx 25 \mu as$, probably all these sources will appear as double sources, and the radio core, obtained from VLBI observations and the optical core obtained by Gaia will not necessarily be the same.

We conclude, remarking that if the inner parts of the accretion disk contain a warp or precess faster than the precession of the outer part, this will produce a very small perturbation that will produce a day-to-month variability of the core flux (Roland et al. 2009).

Acknowledgements. J.R. thanks Alain Lecavelier des Etangs and Simon Prunet for useful discussions and comments. This research has made use of data from the MOJAVE database that is maintained by the MOJAVE team (Lister et al. 2009a) and part of this work was supported by the COST Action MP0905 Black Holes in a Violent Universe.

Appendix A: Fit of component S1 of 1823+568

A.1. Fit of S1 using the precession model

To fit the ejection of component S1, we used 56 observations (28 epochs). We studied the two cases $\pm \omega_0(t - z/V_z)$. The final solution of the fit of component S1 of 1823+568 using a BBH system

![Fig. 15. Using the MOJAVE data (Lister et al. 2009b), we plot the trajectories of C5 and C10. Component C10 is probably ejected by the VLBI core and component C5 is ejected with a large offset from the VLBI core. Components C5 and C10 follow different trajectories and are ejected from different origins, indicating that the nucleus of 3C 279 contains a BBH system. Note that the origin of this caption corresponds to the origin of the ejection of component C5, thus all MOJAVE coordinates have been corrected for the offsets $\Delta W \approx +405 \mu as$ and $\Delta N \approx +110 \mu as$.](image-url)
corresponds to \(+\omega_b(t - z/V_a)\), therefore we discuss only this case in this appendix. In this section, we assume that \(V_a = 0.1c\).

The range of inclination we explore is 0.5\(^\circ\) \(\leq i_o \leq 10^\circ\).

An important parameter for the fit is the time origin of the ejection of the VLBI component. We model the flux using

\[
S_c = \delta_c(t) e^{2\omega_c t}/Fac \\
\times \exp(-T_{\text{opacity}}/t) \times \exp(-t/T_{\text{decay}}),
\]

where \(Fac\) is a scaling factor, \(T_{\text{opacity}}\) is the characteristic time to describe the synchrotron opacity, and \(T_{\text{decay}}\) is the characteristic time to describe the losses. This is the simplest way to model the flux and does not take into account in situ re-acceleration of the relativistic particles along the beam and synchrotron, inverse Compton, or expansion losses. We do not aim to fit the flux light curve, but we wish to compare the time position of the modeled first peak flux with the observed first peak flux (Fig. A.1). This provides the minimum for the time origin of the ejection of the VLBI component. For S1 we find \(i_o \approx 1995.65\). This value agrees well with the time origin obtained from VLBI data interpolation, which is \(i_o \approx 1995.60\) (Glück 2010). This case corresponds to case I, i.e., if there is an offset of the VLBI ejection, it is smaller than or on the order of the smallest error bars of the VLBI component coordinates. In the following we keep \(i_o\) as a free parameter in the range 1995.65 \(\leq i_o \leq 1995.90\).

Because the function \(\chi^2(i_o)\) is mostly flat between 4 and 10 degrees, to continue we arbitrarily adopted the inclination angle \(i_o \approx 6^\circ\). The main results of the fit for the precession model are that

1. the opening angle to the precession cone is \(\Omega \approx 0.46^\circ\);
2. the bulk Lorentz factor of S1 is \(\gamma_c \approx 20\);
3. the origin of S1 is \(i_o \approx 1995.7\); and
4. \(\chi^2(i_o \approx 6^\circ) \approx 67.4\).

### A.2. Determining the family of solutions

The solution is not unique. For the inclination angle previously found, i.e., \(i_o \approx 6^\circ\) and using the parameters of the corresponding solution, we gradually varied \(V_a\) between 0.01c and 0.45c. At each step of \(V_a\), we minimized the function \(\chi^2(\lambda)\), where \(\lambda\) are the free parameters. The function \(\chi^2(V_a)\) remained constant, indicating a degeneracy of the solution, and we obtained the range of possible values for the precession period given in Table A.1.

### A.3. Determining the BBH system parameters

Because the precession is defined by \(+\omega_b(t - z/V_a)\), the BBH system rotation is defined by \(-\omega_b(t - z/V_a)\). In this section, we kept the inclination angle previously found, i.e., \(i_o \approx 6^\circ\) and \(V_a = 0.1c\).

To determine the BBH system parameters corresponding to a value of \(T_p/T_b\), we minimized \(\chi^2(M_1)\) when the mass of the ejecting black hole \(M_1\) varied gradually between 1 \(M_2\) to a value corresponding to \(M_1/M_2 = 2\) with a starting value of \(M_2\), such that \(10^6 \leq M_2 \leq 10^9\). During the minimization \(M_2\) is a free parameter, and at each step of \(M_1\), we minimized the function \(\chi^2(\lambda)\), where \(\lambda\) are the free parameters. Thus we constrained the parameters of the BBH system when the two black holes have the same masses, i.e., \(M_1 = M_2\).

We determined the parameters of the BBH system model for different values of the parameter \(T_p/T_b\), namely \(T_p/T_b = 4.6, 10, 22, 46, 100, \) and 220.

For a given value of \(T_P/T_b\), we found the radius of the BBH system defined by Eq. (8). Note that the radius of the BBH system does not depend on the starting value of \(M_2\).

### A.4. \(\chi^2(T_p/T_b) – \)diagram

In this section, we kept the inclination angle previously found, i.e., \(i_o \approx 6^\circ\), \(V_a = 0.1c\) and assumed \(M_1 = M_2\).

The diagram \(\chi^2(T_P/T_b)\) provides the possible solutions at a given inclination angle. Some of the solutions can be mirage solutions when \(i_o\) varies.

We calculated \(\chi^2(T_p/T_b)\) for 1 \(\leq T_p/T_b \leq 300\). We started for each value of the BBH system parameters found in the previous section, i.e., corresponding to the values of \(T_p/T_b = 4.6, 10, 22, 46, 100, \) and 220, and covered the complete interval 1 \(\leq T_p/T_b \leq 300\). For instance, if we started at \(T_p/T_b = 22\), we covered the ranges varying \(T_p/T_b\) from 22 to 1 and from 22 to 300. We found the possible solutions of the BBH system, i.e., the solutions that correspond to the minima of \(\chi^2(T_P/T_b)\). They are given in Table A.2.

Table 1. Range for the precession period.

<table>
<thead>
<tr>
<th>(V_a)</th>
<th>0.01c</th>
<th>0.45c</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_p(V_a))</td>
<td>(\approx 570000) yr</td>
<td>(\approx 770000) yr</td>
</tr>
</tbody>
</table>

Table 2. Main solutions found for \(i_{o} = 5.98^\circ\).

<table>
<thead>
<tr>
<th>Solution</th>
<th>((T_p/T_b)_{\text{min}})</th>
<th>(\chi^2(\text{min}))</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol 1</td>
<td>(\approx 1.3)</td>
<td>(\approx 50.8)</td>
<td>(\gamma_c &gt; 30)</td>
</tr>
<tr>
<td>Sol 2</td>
<td>(\approx 11.45)</td>
<td>(\approx 53.7)</td>
<td></td>
</tr>
<tr>
<td>Sol 3</td>
<td>(\approx 23.0)</td>
<td>(\approx 63.9)</td>
<td></td>
</tr>
<tr>
<td>Sol 4</td>
<td>(\approx 76)</td>
<td>(\approx 63.6)</td>
<td></td>
</tr>
<tr>
<td>Sol 5</td>
<td>(\approx 109)</td>
<td>(\approx 62.2)</td>
<td>(\gamma_c &gt; 30)</td>
</tr>
</tbody>
</table>

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A.5. Possible offset of the origin of the ejection

In this section, we kept the inclination angle previously adopted, i.e., \( i_o \approx 6^\circ \). We assumed that \( V_a = 0.1c \), \( M_1 = M_2 \), \( T_p/T_b = 11.45 \) and used the parameters of Solution 2 previously found.

To test whether the VLBI component is ejected from the VLBI core or from the second black hole, we calculated \( \chi^2(\Delta W, \Delta N) \), where \( \Delta W \) and \( \Delta N \) are offsets in the west and north directions. The step used in the west and north directions is 5 \( \mu \)as. At each step of \( \Delta W \) and \( \Delta N \), we minimized the function \( \chi^2(\lambda) \), where \( \lambda \) are the free parameters (Fig. A.2). The radius of the BBH system was left free to vary during the minimization.

The minimum of \( \chi^2(\Delta W, \Delta N) \) is \( \approx 49.5 \), and we see from Fig. A.2 that the corresponding non-zero offsets are with \( \Delta N \geq 0.060 \) mas. However, all points with the smallest \( \chi^2(\Delta W, \Delta N) \) are not possible. Indeed, for a point with the smallest \( \chi^2 \), the size of the offset must be equal to the radius of the BBH system calculated at this point. This is the case if the offsets are \( \Delta W_1 \approx +0.010 \) mas and \( \Delta N_1 \approx +0.070 \) mas. The radius of the BBH system at this point is \( R_{\text{bin}} \approx 70 \mu \)as and the offset size is \( \approx 71 \mu \)as, i.e., the offset and the radius of the BBH system are the same at this point.

Therefore we conclude that:

- the VLBI component S1 is not ejected from the VLBI core, but from the second black hole of the BBH system;
- the radius of the BBH system is \( R_{\text{bin}} \approx 71 \mu \)as. It is about twice the smallest error bars of the observed VLBI component coordinates (the component positions), but it is significantly detected (2\( \sigma \) from Fig. A.2).

We must correct the VLBI coordinates from the offset before we continue.

Note that determining the offset of the origin does not depend on the value adopted for the inclination angle. This was shown by calculating the offset at different inclination angles, i.e., \( i_o \approx 4^\circ, 5^\circ, 7^\circ \).

A.6. Determining \( T_p/T_b \)

From this point onward, the original coordinates of the VLBI component S1 are corrected for the offsets \( \Delta W_1 \) and \( \Delta N_1 \) found in the previous section. In this section, we assumed that \( R_{\text{bin}} = 71 \mu \)as, \( M_1 = M_2 \) and \( V_a = 0.1c \).

Previously, we found that Solution 2 is characterized by \( T_p/T_b \approx 11.45 \) for \( i_o \approx 6^\circ \). In this section we obtain the range of possible values of \( T_p/T_b \) when \( i_o \) varies.

We calculated the function \( \chi^2(i_o) \) in the interval \( 2^\circ \leq i_o \leq 7^\circ \), assuming that the ratio \( T_p/T_b \) is free. The relation between \( T_p/T_b \) and \( i_o \) is plotted in Fig. A.3.

Knowing of the possible values of the ratio \( T_p/T_b \) allows us to calculate in the next section the function \( \chi^2(i_o) \) for various values of the ratios \( T_p/T_b \) and \( M_1/M_2 \) and then estimate the mass ratio \( M_1/M_2 \) of the BBH system.

A.7. Preliminary determination of \( i_o \), \( T_p/T_b \) and \( M_1/M_2 \).

In this section, we assumed that \( V_a = 0.1c \) and the radius of the BBH system is \( R_{\text{bin}} \approx 71 \mu \)as.

We varied \( i_o \) between 2 and 7 degrees and calculated \( \chi^2(i_o) \) for various values of \( T_p/T_b \) and \( M_1/M_2 \). The values of \( T_p/T_b \) investigated are \( T_p/T_b \approx 11.45, 8.88, 8.11, 7.76 \), and 7.62. The values of \( M_1/M_2 \) investigated are \( M_1/M_2 = 1.0, 0.5, 0.37, 0.25, \) and 0.1. For each value of \( M_1/M_2 \) we calculated \( \chi^2(i_o) \) for all values of \( T_p/T_b \). The plots \( \chi^2(i_o) \) for \( M_1/M_2 = 1.0 \) and 0.37 are shown in Fig. A.4.

The main results are that:

- when \( i_o \) is larger than about 6 degrees, the bulk Lorentz factor increases and becomes greater than 30, which is excluded;
- the critical value of \( M_1/M_2 \approx 0.5 \);
- if \( M_1/M_2 > 0.5 \), the solution \( \chi^2(i_o) \) is a mirage solution;
- if \( M_1/M_2 < 0.5 \), the solution \( \chi^2(i_o) \) has a minimum;
- the solutions with a robustness larger than 1.7\( \sigma \) are those with \( M_1/M_2 < 0.37 \) (see Table B.1);
- when \( M_1/M_2 \) decreases, the solutions are more robust, but they are of lower quality, i.e., their \( \chi^2 \)(min) increases (see Table B.1); and
- when \( M_1/M_2 < 0.5 \), the value of \( T_p/T_b \) that produces the best fit is \( T_p/T_b \approx 8.88 \), independently of the value of \( M_1/M_2 \).

We present in Table A.3 the results of solutions corresponding to \( T_p/T_b \approx 8.88 \) and \( M_1/M_2 = 0.1, 0.25 \) and 0.37.
Table A.4. The range of $M_1/M_2$ when $T_p/T_b \approx 8.88$.

| $M_1/M_2$ | $i_o$ | $\chi^2$(min) | Robustness($i_o$) |
|-----------|------|-------------|----------------|---|
| 0.09      | $\approx3.79^\circ$ | $\approx51.7$ | $\approx2.4\sigma$ |
| 0.17      | $\approx3.98^\circ$ | $\approx51.2$ | $\approx2.2\sigma$ |
| 0.29      | $\approx4.27^\circ$ | $\approx50.7$ | $\approx1.8\sigma$ |

Table A.5. The range of $M_1/M_2$ when $T_p/T_b \approx 9.88$.

| $M_1/M_2$ | $i_o$ | $\chi^2$(min) | Robustness($i_o$) |
|-----------|------|-------------|----------------|---|
| 0.095     | $\approx4.06^\circ$ | $\approx51.7$ | $\approx2.2\sigma$ |
| 0.16      | $\approx4.22^\circ$ | $\approx51.2$ | $\approx1.8\sigma$ |
| 0.25      | $\approx4.53^\circ$ | $\approx50.7$ | $\approx1.3\sigma$ |

A.8. Determining a possible new offset correction

In this section, we assumed $V_a = 0.1c$. Using the solution found in the previous section (Table B.1), we can verify whether if there is an additional correction to the offset of the origin of the VLBI component. For this, we calculated $\chi^2(\Delta W, \Delta N)$, where $\Delta W$ and $\Delta N$ are offsets in the west and north directions. We assumed the radius of the BBH system to be free to vary. We found that a small additional correction is needed $\Delta W_2 \approx -0.005$ mas and $\Delta N_2 \approx -0.010$ mas.

Finally, we found that the total offset is $\approx60$ mas and the radius of the BBH system is also $R_{\text{bin}} \approx 60$ mas.

A.9. Final fit of component S1 of 1823+568

From this point onward, the coordinates of the VLBI component S1 are corrected for the new offsets $\Delta W_2$ and $\Delta N_2$ found in the previous section. In this section, we assumed $V_a = 0.1c$ and $R_{\text{bin}} = 60$ mas.

We can now find the final solution for S1. We calculated $\chi^2(i_o)$ for various values of $T_p/T_b$ assuming $M_1/M_2 \approx 0.25$. We found that the best range for $T_p/T_b$ is: $8.88 \leq T_p/T_b \leq 9.88$. With this we can estimate the range of the mass ratio assuming $T_p/T_b \approx 8.88$ and $T_p/T_b \approx 9.88$. We defined the range of the mass ratio in the following way:

1. we found the mass ratio that produces a solution of at least $1.7\sigma$ robustness, and
2. we found the mass ratio that produces a solution that is poorer by $1\sigma$ than the previous one, but that is more robust.

The results of the fit are presented in Tables A.4 and A.5. The improvement of the solutions of Tables A.4 and A.5 compared to the solutions of Table A.3 is due to the new offset and the new value of the BBH system radius.

We see that the solutions found with $T_p/T_b \approx 8.88$ are slightly more robust, but both solutions can be used.

The characteristics of the final solution of the BBH system associated with 1823+568 are given in Sect. 4.4.

Appendix B: Fit of component C5 of 3C 279

B.1. Fit of C5 using the precession model

To fit the ejection of component C5 we used 152 observations (76 epochs).

We studied the two cases $\pm60(t - z/V_a)$. The final solution of the fit of component C5 of 3C 279 using a BBH system corresponds to $-\omega_0(t - z/V_a)$, therefore we discuss only this case in this appendix.

To fit the component C5, we assumed $T_d \approx 2000$ yr. In this section, we assume that $V_a = 0.1c$.

The range of inclination explored is $0.5^\circ \leq i_o \leq 1^\circ$.

To begin, we allowed the time origin of the VLBI component to be a free parameter. We assumed $1997.0 \leq t_o \leq 1998.5$. We found that the function $\chi^2(i_o)$ is characteristic of a function corresponding to case II (see Sect. 5.1), and the possible range for the inclination angle is $[0.5, 5.5]$. The time origin is $t_o \approx 1997.52$ when $i_o \to 0.55$, and the time origin is $t_o \approx 1998.15$ when $i_o \to 5.5$. We plotted the first peak flux corresponding to the solution $t_o \approx 1998.15$ and $i_o \to 5.5$ (solution with the smallest $\chi^2$ and found that it is too early by at least eight months (dash line in Fig. B.2). As indicated in Sect. A.1, we do not aim to fit the flux light curve, but we wish to compare the time position of the modeled first peak flux with the observed first peak flux (Fig. B.2).
The possible range for the inclination angle is reduced to 0° ≤ i₀ ≤ 4.3°. We assumed that the time origin is 1998.0. It stops at i₀ ≈ 4.3° and i₀ ≈ 0.8° because at these points the bulk Lorentz factor becomes larger than 30. Bottom figure: the bulk Lorentz factor diverges when the bulk Lorentz factor becomes larger than 30.

We concluded that the minimum time for the ejection of C5 is t₁ ≈ 1997.80. Now using the precession model, we calculated χ²(V₀) to be a free parameter in the range 0 ≤ V₀ ≤ 0.45c. The function χ²(V₀) remains constant, indicating a degeneracy of the solution, and we obtained the range of possible values for the precession period given in Table B.1.

Table B.1. Range for the precession period.

<table>
<thead>
<tr>
<th>V₀</th>
<th>0.01c</th>
<th>0.45c</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₁(V₀)</td>
<td>≈91,900 yr</td>
<td>≈1150 yr</td>
</tr>
</tbody>
</table>

Next, we allowed t₀ to be a free parameter in the range 1998.80 ≤ t₀ ≤ 1999.10 and calculated the new function χ²(i₀). The possible range for the inclination angle is reduced to 0.8° ≤ i₀ ≤ 4.3°. The plots of χ²(i₀) and γ(i₀) are presented in Fig. B.1. We plotted the first peak flux corresponding to the solution t₀ ≈ 1998.80 and i₀ → 4.3° (solid line in Fig. B.2). From Fig. B.2, we conclude that the minimum time for the ejection of C5 is t₀ ≥ 1998.80.

The behavior of the functions χ²(i₀) and γ(i₀) are the second signature of case II, i.e., the offset is larger than the smallest error bars of the VLBI component coordinates. We see from Fig. B.1 that the bulk Lorentz factor is γ ≥ 22. Because the function χ²(i₀) does not show a minimum, we arbitrarily chose an inclination angle such that 22 ≤ γ ≤ 26 and the corresponding χ²(i₀) is the smallest. To continue, we chose i₀ ≈ 2.98° and the corresponding parameters of the precession solution (the χ² of this solution is χ² ≈ 1211 and its bulk Lorentz factor is γ ≈ 22.6). We used this solution to apply the method explained in Sect. 3 and we will see in the following how the BBH system model allows us to find the concave part of the function χ²(i₀).

B.2. Determining the family of solutions (precession model)

The solution is not unique. For the inclination angle previously found, i.e., i₀ ≈ 2.98° and using the parameters of the corresponding solution, we gradually varied V₀ between 0.01c and 0.45c. The function χ²(V₀) remains constant, indicating a degeneracy of the solution, and we obtained the range of possible values for the precession period given in Table B.1.

B.3. Possible offset of the origin of the ejection (precession model)

In this section, we kept the inclination angle previously found, i.e., i₀ ≈ 2.98°. We assumed that V₀ = 0.1 and used the parameters of the solution previously found.

To test whether the VLBI component is ejected from the VLBI core or if it is ejected with an offset of the origin, we calculated χ²(ΔW, ΔN), where ΔW and ΔN are offsets in the west and north directions, using the precession model. The step used in West and North directions is 10 μas.

We see from Fig. B.3, that non-zero offsets are possible and the smallest offsets of the coordinates are ΔW ≈ +0.300 mas and ΔN ≈ +0.280 mas, which, a priori, corresponds to an offset of the space origin of ≥410 μas or to a BBH system of radius R₉₁₁ ≈ 410 μas. This minimum offset corresponds to an improvement of about 28σr. If the offset of the space origin can be estimated using the precession model, it cannot be explained if we assume that the nucleus contains a single black hole, but it can be explained if we assume that the nucleus contains a BBH system.

It is important to note that the offset does not depend on the inclination angle chosen in Sect. B.1. Indeed, we took the solution corresponding to i₀ ≈ 1.5°, whose χ² is χ² ≈ 1610 and whose bulk Lorentz factor is γ ≈ 24, and we calculated χ²(ΔW, ΔN), which yielded the same result.

It is easy to prove that the value of the offset of the space origin is related to the time origin problem. Indeed, Fig. B.3 shows that there is a significant offset of the space origin when we assume that the time origin of component C5 of 3C 279 is t₀ ≥ 1998.80. Now, using again the precession model, we...
corresponds to an ejection that is about seven months too early. At this point, the time origin is a free parameter (Fig. B.4). We see from Fig. B.3.

To continue, two possibilities arise:

1. either we keep the original VLBI coordinates and determine the parameters of the BBH system and the \( \chi^2(T_p/T_b) \) – diagram. Then, we determine a first offset correction using the BBH model, and after a preliminary determination of \( T_p/T_b \) and \( M_1/M_2 \), we determine a second offset correction using the BBH model;

2. or we apply the precession offset correction to the VLBI coordinates and then we determine the parameters of the BBH system and the \( \chi^2(T_p/T_b) \) – diagram. Then we determine a first offset correction using the BBH model, and after a preliminary determination of \( T_p/T_b \) and \( M_1/M_2 \), we determine a second offset correction using the BBH model.

For component C5 of 3C 279, the two possibilities were followed. We found that they provide the same result in the end.

In this article, we present the first one.

The determination of the offsets of the origin of the ejection does not depend on the inclination angle.

### B.4. Determining the BBH system parameters

Because the precession is defined by \(-\omega_p(t - z/V_c)\), the BBH system rotation is defined by \(+\omega_p(t - z/V_c)\). In this section we kept the inclination angle previously found, i.e., \( i_0 \approx 2.98^\circ \) and \( V_c \approx 0.1c \).

In the previous section, we saw that the BBH system has a large radius, i.e., \( R_{bin} \geq 410 \mu as \). Therefore, we determined the parameters of a BBH system with small \( T_p/T_b \) and a radius for the BBH system that is a free parameter (solutions with small \( T_p/T_b \) have large radii), i.e., we determined the parameters of a BBH system with \( T_p/T_b = 1.01 \) and calculated the corresponding \( \chi^2(T_p/T_b) \) – diagram.

### B.5. \( \chi^2(T_p/T_b) \) – diagram

In this section, we kept the inclination angle previously found, i.e., \( i_0 \approx 2.98^\circ \), \( V_c = 0.1c \) and assumed \( M_1 = M_2 \). Furthermore we assumed that the radius of the BBH system is a free parameter.

We calculated \( \chi^2(T_p/T_b) \) for \( 1 \leq T_p/T_b \leq 300 \). We started for BBH system parameters corresponding to the values of \( T_p/T_b = 1.01 \) and cover the complete interval of \( T_p/T_b \). The result is shown in Fig. B.5.

We found the one solution given in Table B.2.

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**Fig. B.3.** Using the precession model, we calculated \( \chi^2(\Delta W, \Delta N) \) corresponding to the solution with \( i_0 = 2.98^\circ \). Non-zero offsets are possible and the smallest offsets are \( \Delta W \approx 100 \mu as \) and \( \Delta N \approx 150 \mu as \). At this point, the time origin is \( t_o \approx 1998.45 \) which is \( \approx 7 \) months too early.

**Fig. B.4.** Calculation of \( \chi^2(\Delta W, \Delta N) \) using the precession model and assuming that the time origin is a free parameter. We find that non-offset are possible and the smallest offset corresponds to the point \( \Delta W \approx 100 \mu as \) and \( \Delta N \approx +150 \mu as \). At this point, the time origin is \( t_o \approx 1998.45 \) which is \( \approx 7 \) months too early.

**Fig. B.5.** Calculation of \( \chi^2(T_p/T_b) \). The curve corresponds to the minimization when \( T_p/T_b \) varies from 1 to 300. There is one solution S1.
Table B.2. Solution found for \( i_o \approx 2.98^\circ \).

<table>
<thead>
<tr>
<th>Solution</th>
<th>( (T_p/T_b)_{\min} )</th>
<th>( \chi^2(\text{min}) )</th>
<th>( R_{\text{bin}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>( \approx 2.13 )</td>
<td>( \approx 453 )</td>
<td>( \approx 389 \mu\text{as} )</td>
</tr>
</tbody>
</table>

Fig. B.6. Calculation of \( \chi^2(\Delta W, \Delta N) \) using the BBH model. Contour levels are 246, 247, 250, 255, etc., corresponding to the minimum, 1\( \sigma \), 2\( \sigma \), 3\( \sigma \), etc. There is a valley of possible offsets, but the size of the offset must be the same as the radius of the BBH system. This is true when the offsets are \( \Delta W_1 \approx +0.490 \mu\text{as} \) and \( \Delta N_1 \approx 0.005 \mu\text{as} \).

B.6. Determining the offset of the origin of the ejection (BBH model)

In this section, we kept the inclination angle previously found, i.e., \( i_o \approx 2.98^\circ \). We assumed that \( V_a = 0.1c \), \( M_1 = M_2 \).

We calculated \( \chi^2(\Delta W, \Delta N) \), where \( \Delta W \) and \( \Delta N \) are offsets in the west and north directions. The step used in the west and north directions is 5 \( \mu\text{as} \). The radius of the BBH system and \( T_p/T_b \) are free parameters during the minimization.

We calculated \( \chi^2(\Delta W, \Delta N) \) starting with the parameters of solution S1 found in the previous section. The result is shown in Fig. B.6.

We see from Fig. B.6 that non-zero offsets are possible. However, all points with the smallest \( \chi^2(\Delta W, \Delta N) \) are not possible. Indeed, for a point with the smallest \( \chi^2 \), the offset size must be equal to the radius of the BBH system calculated at this point. This is the case if the offsets are \( \Delta W_1 \approx +0.490 \mu\text{as} \) and \( \Delta N_1 \approx +0.005 \mu\text{as} \).

The radius of the BBH system at this point is \( R_{\text{bin}} \approx 487 \mu\text{as} \) and the offset size is \( \approx 490 \mu\text{as} \), i.e., the offset and the radius of the BBH system are the same at this point.

Therefore we conclude that

- the VLBI component C5 is not ejected from the VLBI core, but from the second black hole of the BBH system, and
- the radius of the BBH system is \( R_{\text{bin}} \approx 490 \mu\text{as} \). It is more than ten times the smallest error bars of the VLBI component coordinates.

Note that if the size of the offset found with the BBH model is the same as the size of the offset found with the precession model, the first offsets are not the same for the coordinates. However, after the preliminary determination of the ratios \( T_p/T_b \) and \( M_1/M_2 \), the second and third offset corrections provide the same final offset corrections (the two methods indicated in Sect. B.3 provide the same corrections in the end).

B.7. Preliminary determination of \( i_o, T_p/T_b \) and \( M_1/M_2 \)

From this point onward, the original coordinates of the VLBI component C5 are corrected for the offsets \( \Delta W_1 \) and \( \Delta N_1 \) found in the previous section. In this section, we assumed that \( V_a = 0.1c \) and the radius of the BBH system is \( R_{\text{bin}} = 490 \mu\text{as} \).

For given values of the ratio \( M_1/M_2 = 1.0, 1.25, 1.50, 1.75, \) and 2.0, we varied \( i_o \) between 3.0 and 10 degrees and calculated \( \chi^2(i_o) \) assuming that the ratio \( T_p/T_b \) is variable.

We found that \( \chi^2(i_o) \) is minimum for the parameters

- \( i_o \approx 5.9^\circ \);
- \( M_1/M_2 \approx 1.75 \); and
- \( T_p/T_b \approx 14.6 \).

B.8. Determining a possible new offset correction

In this section, we assumed \( V_a = 0.1c \).

With \( i_o = 5.9^\circ \), with a variable ratio \( T_p/T_b, M_1/M_2 \approx 1.75 \) and the parameters of the solution found in the previous section, we can verify whether there is an additional correction to the offset of the origin of the VLBI component. We calculated \( \chi^2(\Delta W, \Delta N) \), where \( \Delta W \) and \( \Delta N \) are offsets in the west and north directions. We assumed that the radius of the BBH system is left free to vary. The result is shown in Fig. B.7. We found that an additional correction is needed, namely \( \Delta W_2 \approx -0.085 \mu\text{as} \) and \( \Delta N_2 \approx +0.105 \mu\text{as} \).

At this point the total offset is \( \approx 418 \mu\text{as} \) and the radius of the BBH system is \( R_{\text{bin}} \approx 420 \mu\text{as} \).
The determination of the solution used an iterative method. Starting with a given value of \( T_p/T_b \), we calculated \( \chi^2(i_o) \) for various values of \( M_1/M_2 \). Then we calculated for the parameters corresponding to the solution found the function \( \chi^2(T_p/T_b) \) to determine the new value of \( T_p/T_b \) that minimizes the function \( \chi^2(T_p/T_b) \). Starting with the new value of \( T_p/T_b \), we repeated the procedure.

At each step of the procedure, we calculated \( \chi^2(\gamma) \) to check that the solution corresponds to the concave part and is not a mirage solution.

The best fit is obtained for \( T_p/T_b \approx 140 \) and \( M_1/M_2 \approx 2.75 \). The results of the fits are presented in Table B.3.

We plot in Fig. B.9 the calculation of \( \chi^2(\gamma) \) corresponding to the solution characterized by \( T_p/T_b = 140 \) and \( M_1/M_2 = 2.75 \). It shows that the solution is not a mirage solution in relation to \( \gamma \).

The best fit is obtained for \( T_p/T_b \approx 140 \) (see Fig. B.10). When the ratio \( T_p/T_b \) increases, the \( \chi^2 \) remains mostly constant but the robustness of the solution in relation to \( \gamma \) increases.

Finally, we plot in Fig. B.11 the function \( \chi^2(i_o) \).

The characteristics of final solution of the BBH system associated with 3C 279 are given in Sect. 5.4.

### Appendix C: Error bars

#### C.1. Minimum error bar values

Observations used to fit the components S1 of 1823+568 and C5 of 3C 279 were performed at 15 GHz. We adopted for the
minimum values of the error bars, \( \Delta_{\text{min}} \), values in the range \( \text{Beam}/15 \leq \Delta_{\text{min}} \leq \text{Beam}/12 \).

There are three important points concerning the minimum values used for the error bars:

1. The minimum values are chosen empirically, but the adopted values are justified a posteriori by comparing of the value of \( \chi^2 \) of the final solution and the number of constraints used to make the fit. Indeed, the reduced \( \chi^2 \) has to be close to 1.
2. The minimum value of the error bars used at 15 GHz produces a value of \( \chi^2 \) final consistent with the value of the realistic error obtained from the VLBI Service for Geodesy and Astrometry (Schlüter & Behrend 2007), which is a permanent geodetic and astrometric VLBI program. It has been monitoring the position of thousands of extragalactic radio sources for more than 30 years. In 2009, the second realization of the International Celestial Reference Frame (ICRF2) was released (Fey et al. 2009), obtained after the treatment of about 6.5 millions of ionosphere-corrected VLBI group delay measurements at 2 and 8 GHz. This catalog is currently the most accurate astrometric catalog, giving absolute positions of 3414 extragalactic bodies at 8 GHz. The observations at 2 GHz are used for the ionospheric correction only. Therefore, the positions at 2 GHz are not provided. The ICRF2 is found to have a noise floor of the minimum values of the error bars suggested by Lister & Homan (2005), i.e. the value for the range of phase noise lower than 40 \( \mu \)as.

Since the ICRF2 release, the positional accuracy of the sources has increased, and it is likely that the next VLBI realization of the ICRF will have a noise floor lower than 20 \( \mu \)as.

3. The adopted minimum value of the error bars also includes typical errors due to tau, effects, which shift the measured position at different frequencies (Lobanov 1998).

Thus the minimum values for the error bars adopted at 15 GHz, using Eq. (19), are correct. The fit of VLBI coordinates of components of 3C 345 (work in progress) indicates that the adopted values for the minimum values of the error bars, using Eq. (19), are correct for frequencies between 8 GHz and 22 GHz. At lower frequencies, the minimum values may be higher than \( \text{Beam}/12 \) due to strong opacity effects and at 43 GHz, the minimum values are also probably higher (\( \approx 20 \mu \)as).

It has been suggested by Lister & Homan (2005) that the positional error bars should be about 1/5 of the beam size. To study the influence of the minimum values of the error bars on the characteristics of the solution, we calculate in the next sections the solution of the fit of the component C5 assuming for the minimum values of the error bars the value suggested by Lister & Homan (2005), i.e. the value \( \Delta_{\text{min}} = \text{Beam}/5 \), or (\( \Delta W \)) \( \approx 102 \mu \)as and (\( \Delta N \)) \( \approx 267 \mu \)as.

C.2. Fit of C5 using the precession model

We look for a solution with \(- \omega \tau (t - z/V_x)\) and \( \tau \geq 1980.80 \) (see Sect. B.1).

In this section, we assumed that \( V_x = 0.1 c \).

The range of inclination explored is \( 0^\circ \leq \iota \leq 10^\circ \).

We allowed \( \iota \) to be a free parameter in the range \( 1998.80 \leq \iota \leq 1999.10 \) and calculated the function \( \chi^2(\iota) \). The possible range for the inclination angle is \( 0.7^\circ \leq \iota \leq 4.3^\circ \). The plots of \( \chi^2(\iota) \) and \( \gamma(\iota) \) are presented in Fig. C.1.

The behavior of the functions \( \chi^2(\iota) \) and \( \chi(\iota) \) are the second signature of case II.

Comparison of Figs. C.1 and B.1 shows that the range of the inclination angle, and the values of the bulk Lorentz factor in this range, are the same for the different values of the minimum error bars used.

C.3. Possible offset of the origin of the ejection (precession model)

In this section, we keep the inclination angle used in Sect. B.3, i.e., \( \iota = 3.3^\circ \). We assumed that \( V_z = 0.1 \) and used the parameters of the solution found in Sect. C.2.

We calculated \( \chi^2(\Delta W, \Delta N) \), where \( \Delta W \) and \( \Delta N \) are offsets in the west and north directions, using the precession model. The step used in the west and north directions is 10 \( \mu \)as. The result of the calculation is plotted in Fig. C.2.

Comparison of Fig. C.2 with Fig. B.3 shows that

- the smallest offsets of the coordinates are \( \Delta W \approx +0.320 \) mas and \( \Delta N \approx +0.280 \) mas. They are similar to the offsets found assuming that the minimum error bars are \( \Delta_{\text{min}} = \text{Beam}/15 \) (see Sect. B.3),
- the value of \( \chi^2 \) at the minimum is \( \chi^2_{\text{min}} \approx 67 \) instead of \( \chi^2_{\text{min}} \approx 400 \) when the minimum error bars are \( \Delta_{\text{min}} = \text{Beam}/15 \). The reduced \( f \) is \( \chi^2 = 67/152 \approx 0.44 \), indicating that the minimum error bars are too large.
Table C.1. Solution found for $i_0 \approx 2.98^\circ$.

<table>
<thead>
<tr>
<th>Solution</th>
<th>$(T_p/T_b)_{\text{min}}$</th>
<th>$\chi^2(\text{min})$</th>
<th>$R_{\text{bin}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sol 1</td>
<td>$2.24$</td>
<td>$64.6$</td>
<td>$398 \mu$as</td>
</tr>
</tbody>
</table>

Accordingly, with high values for the minimum values of the error bars, we find using the precession model that the component C5 is ejected with an offset of the space origin of at least $0.425$ mas with a robustness higher than $11\sigma$. The offset of the space origin can be estimated using a single black hole and the precession of the accretion disk. It cannot be explained when we assume that the nucleus contains a single black hole, but it can be explained when we assume that the nucleus contains a BBH system.

C.4. $\chi^2(T_p/T_b) –$ diagram

Because the precession is defined by $-\omega_b(t-z/V_o)$, the BBH system rotation is defined by $+\omega_b(t-z/V_o)$. As in Sect. B.4, we calculated the BBH parameters for the inclination angle $i_0 \approx 2.98^\circ$ and the ratio $T_p/T_b = 1.01$ and calculated the corresponding $\chi^2(T_p/T_b)$ – diagram assuming $M_1 = M_2$ and $V_o = 0.1c$.

We calculated $\chi^2(T_p/T_b)$ for $1 \leq T_p/T_b \leq 300$. The result is shown in Fig. C.3.

We found a possible solution of the BBH system given in Table C.1.

Comparison of Tables B.2 and C.1 and of Figs. B.5 and C.3 shows that the solutions S1 and Sol1 are mostly identical.

C.5. Determining the offset of the origin of the ejection (BBH model)

In this section, we kept the inclination angle previously found, i.e., $i_0 \approx 2.98^\circ$. We assumed that $V_o = 0.1c$, $M_1 = M_2$.

We calculated $\chi^2(\Delta W, \Delta N)$, where $\Delta W$ and $\Delta N$ are offsets in the west and north directions. The step used in the west and north directions is $5 \mu$as. The radius of the BBH system and $T_p/T_b$ are free parameters during the minimization.

We calculated $\chi^2(\Delta W, \Delta N)$ starting with the parameters of solution Sol 1 found in the previous section. The result is shown in Fig. C.4.

We see from Fig. C.4 that non-zero offsets are possible. However, all points with the smallest $\chi^2(\Delta W, \Delta N)$ are not possible. Indeed, for a point with the smallest $\chi^2$, the size offset must be equal to the radius of the BBH system calculated at this point. This is the case if the offsets are $\Delta W_1 \approx +0.495$ mas and $\Delta N_1 \approx +0.005$ mas.

The radius of the BBH system at this point is $R_{\text{bin}} \approx 495 \mu$as and the offset size is $\approx 495 \mu$as, i.e. the offset and the radius of the BBH system are the same at this point.
The BBH system is free to vary. The result is shown in Fig. C.5. We found that the radius of the BBH system is \( R \approx 419 \mu\text{as} \).

Comparison of the result found in Sect. B.6 and of Figs. C.4 and B.6 show that the offset determined using large error bars is the same as the offset calculated with the small error bars.

Therefore we conclude that the VLBI component C5 is not ejected from the VLBI core but from the second black hole of the BBH system.

C.6. Determining a possible new offset correction

From this point onward, the original coordinates of the VLBI component C5 are corrected for the offsets \( \Delta W_i \) and \( \Delta N_i \) found in the previous section.

In this section, we assumed \( V_a = 0.1c \).

As in Sect. B.7, we preliminarily determined the parameters \( T_{p}/T_b, M_1/M_2 \) and \( i_0 \).

With \( i_0 \approx 5.9^\circ \), using the ratios \( T_{p}/T_b \) free and \( M_1/M_2 \approx 1.75 \), we can verify whether there is an additional correction to the offset of the origin of the VLBI component. We calculated \( \chi^2(\Delta W, \Delta N) \), where \( \Delta W \) and \( \Delta N \) are offsets in the west and north directions. We assumed that the radius of the BBH system is set free to vary. The result is shown in Fig. C.5. We found that an additional correction is needed, namely \( \Delta W_2 \approx -0.085 \text{mas} \) and \( \Delta N_2 \approx +0.085 \text{mas} \).

At this point the total offset is \( 419 \mu\text{as} \) and the radius of the BBH system is \( R_{\text{bb}} \approx 419 \mu\text{as} \).

Thus, using for the highest values of the error bars the values \( \Delta_{\text{min}} = Beam/5 \), we found that the final offset is \( \Delta W_i \approx +0.410 \mu\text{as} \) and \( \Delta N_i \approx +0.090 \mu\text{as} \) and the radius of the BBH system is \( R_{\text{bb}} \approx 0.419 \mu\text{as} \).

These values have to be compared with the values obtained assuming for the lowest error bars the value used \( \Delta_{\text{min}} = Beam/15 \), which are \( \Delta W_i \approx +0.405 \mu\text{as} \) and \( \Delta N_i \approx +0.110 \mu\text{as} \) and the radius of the BBH system is \( R_{\text{bb}} \approx 0.420 \mu\text{as} \).

C.7. Final solution

The characteristics of the final solution determined assuming for the minimum value of the error bars the value \( \Delta_{\text{min}} = Beam/5 \) are

- \( T_{p}/T_b \approx 140; \)
- \( M_1/M_2 \approx 2.75; \)
- \( i_0 \approx 11.0^\circ; \) and
- \( \chi^2 \approx 28.9. \)

Thus the reduced \( \chi^2 \) at the minimum is \( \chi^2_{r} \approx 0.19 \), indicating that the minimum error bars are too large.

C.8. Conclusion

We determined the characteristics of the solution, assuming for the minimum value of the error bars the value \( \Delta_{\text{min}} = Beam/5 \) suggested by Lister & Homan (2005). We found that

1. the characteristics of the solution are the same as those of the solution determined assuming for the minimum value of the error bars the value \( \Delta_{\text{min}} = Beam/15 \), and
2. the corresponding reduced \( \chi^2 \) is \( \chi^2_{r} \approx 28.9/152 \approx 0.19 \), indicating that the minimum error bars are too large.

The correct value for the minimum error bars at 15 GHz is \( Beam/15 \leq \Delta_{\text{min}} \leq Beam/12 \).

This value

1. produces a reduced \( \chi^2, \chi^2_{r} \approx 1; \)
2. the minimum value agrees with the value of the realistic error obtained from the VLBI Service for Geodesy and Astrometry (Schlüter & Behrend 2007), and
3. the fit of VLBI coordinates of components of 3C 345 (work in progress) indicates that the adopted values for the minimum values of the error bars, i.e., \( Beam/15 \leq \Delta_{\text{min}} \leq Beam/12 \), are correct for frequencies between 8 GHz and 22 GHz.

References

Glück, C. B. 2010, in Diplomarbeit, Universität zu Köln