Tensorial depolarization of alkali atoms by isotropic collisions with neutral hydrogen
(Research Note)

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ABSTRACT

We consider the problem of isotropic collisions between an alkali atom and neutral hydrogen. We calculate the collisional tensorial components of general p and s-states, characterized by their effective principal quantum number \( n^* \). We find that the behaviour of the tensorial components obey simple power laws allowing quick calculations of the depolarizing collisional rates. As a possible application, our results should allow a rigorous treatment of the atomic polarization profiles of the D1–D2 lines of alkali atoms. Finally, we notice that important problems remain unresolved, such as the role of collisions in the Paschen-Back conditions.

Key words. scattering – polarization – atomic processes – Sun: atmosphere – line: formation

1. Context, notations, and numerical results

1.1. Context

Atomic polarization consists in an imbalance in the populations of the Zeeman sublevels pertaining to a given atomic level. The Zeeman sublevels are coherences between the sublevels themselves. The scattering polarization is the observational signature of the atomic polarization (e.g. Landi Degl’Innocenti & Landolfi 2004; Trujillo Bueno 2001). In real plasmas such as the solar atmosphere, scattering polarization is usually rather small in problems of practical interest (e.g. Fano 1963; Omont 1977; Sahal-Bréchot 1977; Landi Degl’Innocenti & Landolfi 2004). It is useful to note that for isotropic processes, the coupling terms implying \( k \neq k' \) and a transfer of coherence \( q \) to \( q' \) are zero, and that the collisional depolarization and polarization transfer rates are \( q \)-independent. At the solar photosphere where the second solar spectrum is formed, the non-diagonal components of the collisional depolarization matrix are zero.

Scattering polarization is sensitive to radiation, to magnetic and electric fields, to collisions, etc. As the scattering polarization is usually rather small in problems of practical interest (e.g. scattering polarization of the order of 1% in Fraunhofer lines in the photosphere and chromosphere of the Sun), careful and precise modelling of that polarization is of fundamental importance for learning in particular about a weak and even unresolved magnetic field by its Hanle effect. For this purpose, it is of interest to obtain detailed statistical equilibrium equations (SEE) including all the tensorial components of the collisional rates.

1.2. Notations

Let us consider the atomic states \((\alpha j)\) of alkali atoms where \(j\) is the total angular momentum and \(\alpha\) represents the other quantum numbers necessary to define the electronic level. The atomic states \((\alpha j)\) are quantified by the spherical tensor components \(\rho_k^{\alpha}(\alpha j)\) of the density matrix, \(k\) is the tensorial order, and \(q\) represents the coherences between the levels (e.g. Fano 1963; Omont 1977; Sahal-Bréchot 1977; Landi Degl’Innocenti & Landolfi 2004). It is useful to note that for isotropic processes, the coupling terms implying \( k \neq k' \) and a transfer of coherence \( q \) to \( q' \) are zero, and that the collisional depolarization and polarization transfer rates are \( q \)-independent.

In the case of collisions of neutral atoms with neutral hydrogen, the inelastic and super-elastic excitation between two different electronic levels are negligible. Therefore, the index \(\alpha\) is omitted from now on for the sake of brevity. We apply the general theory developed by Derouich et al. (2003, 2005) and the associated numerical code to provide the all non-zero tensorial collisional rates of any \( s \) and \( p \) level. We denote by \( C^{\alpha}_{k}(j) \) the collisional rates due to elastic collisions and by \( D^{\alpha}_{k}(j) = C^{\alpha}_{k}(j) - C^{\alpha}_{k}(j) \) the usual depolarizing rate; the expression of \( D^{\alpha}_{k} \) is given for example by Eqs. (7) and (9) of Derouich et al. (2003). \( C^{\alpha}_{k}(j,j) \) and \( C^{\alpha}_{k}(j,j_u) \) are the collisional transfer rates due to inelastic and super-elastic collisions, respectively (see Derouich et al. 2007; Landi Degl’Innocenti & Landolfi 2004). The indices \( l \) and \( u \) denote any level of energy, respectively, lower or higher than the energy of the level \( j \). The contribution of isotropic collisions in the SEE is given by Eq. (5) of Derouich et al. (2007) and Eq. (7.101) of Landi Degl’Innocenti & Landolfi (2004).

1.3. Numerical results

The present work provides new complementary numerical results indispensable for a rigorous analysis of the polarization...
profiles. Derouich et al. (2003) determined solely the depolarization rates $D^{k=2}$ of P-states. Table 4 of Derouich et al. (2003) gives the alignment ($k = 2$) depolarization rates as a function of $n^*$ for the temperatures of 5000 K and 6000 K. The collisional rates $C^E_k$, $C^k_j$, and $C^S_j$ were not given. This paper attempts to fill this lacuna by providing $C^E_k$, $C^k_j$, and $C^S_j$ as a power function of $n^*$ and $T$. In addition, compact analytical power-laws of the $C^E_k$ rates associated with S-states are derived from the collisional method presented in Derouich et al. (2005).

Grids of collisional rates are computed for the effective principal quantum number $n^*$ of the s and p-states in the interval [1.5, 3]. We adopt a step size of 0.1 and we derive laws describing the behaviours of the various parameters with $n^*$, which are, with correlation coefficients $R > 0.9$, fit by

- **Spherical tensor components of the level $3s \ ^2S \ _{1/2}$**

$$C^E_k \left( \frac{1}{2} \right) = 1.838 \times 10^{-9} n_H \times n^{0.675} \left( \frac{T}{5000} \right) 0.416$$

$$C^I_k \left( \frac{1}{2} \right) = 0.958 \times 10^{-9} n_H \times n^{-0.011} \left( \frac{T}{5000} \right) 0.416$$

(1)

- **Spherical tensor components of the level $3p \ ^2P \ _{1/2}$**

$$C^E_k \left( \frac{1}{2} \right) = 1.301 \times 10^{-9} n_H \times n^{1.235} \left( \frac{T}{5000} \right) 0.406$$

$$C^I_k \left( \frac{1}{2} \right) = 0.753 \times 10^{-9} n_H \times n^{-1.085} \left( \frac{T}{5000} \right) 0.420$$

(2)

- **Spherical tensor components of the level $3p \ ^2P \ _{3/2}$**

$$C^E_k \left( \frac{3}{2} \right) = 1.685 \times 10^{-9} n_H \times n^{2.671} \left( \frac{T}{5000} \right) 0.420$$

$$C^I_k \left( \frac{3}{2} \right) = 0.953 \times 10^{-9} n_H \times n^{2.733} \left( \frac{T}{5000} \right) 0.390$$

$$C^E_k \left( \frac{3}{2} \right) = 0.472 \times 10^{-9} n_H \times n^{2.400} \left( \frac{T}{5000} \right) 0.380$$

$$C^I_k \left( \frac{3}{2} \right) = 0.482 \times 10^{-9} n_H \times n^{2.670} \left( \frac{T}{5000} \right) 0.400$$

$$C^I_k \left( \frac{3}{2} \right) = 0.975 \times 10^{-9} n_H \times n^{2.705} \left( \frac{T}{5000} \right) 0.400$$

$$C^I_k \left( \frac{3}{2} \right) = -0.095 \times 10^{-9} n_H \times n^{2.545} \left( \frac{T}{5000} \right) 0.410$$

(3)

Only the excitation collisional transfer rates $C^E_k(\frac{3}{2}, \frac{1}{2})$ and $C^k_j(\frac{3}{2}, \frac{1}{2})$ are given. However, it is straightforward to retrieve the values of the deexcitation collisional rates $C^S_j(\frac{3}{2}, \frac{1}{2})$ and $C^S_j(\frac{3}{2}, \frac{1}{2})$ by applying the detailed balance relation

$$C^S_j(j, j_0) = \frac{2j + 1}{2j_0 + 1} \exp \left( \frac{E_{j_0} - E_j}{k_B T} \right) C^E_k(j_0, j)$$

(4)

with $E_j$ being the energy of the level $(j)$ and $k_B$ the Boltzmann constant.

We note that all the collisional rates are given in $s^{-1}$, $n_H$ is the neutral hydrogen density in cm$^{-3}$, and the temperature $T$ is in Kelvin. Table 1 gives the effective principal quantum number $n^*$ of the s and p-states of the alkali atoms Li I, Na I, K I, Rb I, and Cs I. In particular, since $n^* = 1.627$ for the ground state of Na I, the destruction of the orientation rate ($k = 1$) is

$$D^1 \left( \frac{1}{2} \right) = C^E_k \left( \frac{1}{2} \right) - C^I_k \left( \frac{1}{2} \right)$$

$$= 1.838 \times n^{0.675} - 0.958 \times n^{-0.011} \times 10^{-9} n_H \left( \frac{T}{5000} \right) 0.416$$

$$= 4.246 \times 10^{-9} n_H \times \left( \frac{T}{5000} \right) 0.416$$

(5)

which is very close to the fully quantum rate given in the erratum of Kerkeni et al. (2000), who at, $T = 5000$ K, found a rate of $4.32 \times 10^{-9} n_H$ $s^{-1}$.

2. **Hyperfine structure**

In typical solar conditions, the hyperfine splitting is usually much lower than the inverse of the typical time duration of a collision, hence one can assume that the nuclear-spin is conserved during the collision. This is the frozen nuclear spin approximation implying that the hyperfine collisional rates are given as a linear combination of the fine rates $C^E_k$, $C^k_j$, and $C^S_j$ given in this present paper (e.g. Nienhuis 1976; Omont 1977).

3. **Conclusion**

Tensorial collisional components are given in this work and could be implemented in the numerical simulations of the scattering polarization in a way similar to radiative rates. Hyperfine and fine collisional rates may be derived from simple power laws provided in this work.

Our results are valid for collisions without an external magnetic field or for a sufficiently weak field. Under solar conditions, to the best of the author’s knowledge, there has been no study that includes magnetic fields explicitly in the calculations of collisional rates. Depolarizing collisional rates commonly encountered in the literature should be used in the Hanle effect regime but are not valid in the Paschen-Back effect regime. It remains a challenge to take into account the effect of the magnetic fields in the calculation of the collisional coefficients.

References


Omont, A. 1977, Prog. Quantum Electron., 5, 69


\(^1\) It is important, however, not to confuse this condition with the fact that the SEE must be solved for the hyperfine levels when the inverse of the lifetime of the level is lower than the hyperfine splitting, i.e. the hyperfine levels are separated.