Electron-cyclotron maser emission by power-law electrons in coronal loops

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ABSTRACT

Context. The electron-cyclotron maser (ECM) instability is an important mechanism that amplifies electromagnetic radiation directly by nonthermal electrons trapped in magnetic fields. The nonthermal electrons frequently have a negative power-law distribution with a lower energy cutoff ($E_\text{c}$), which will depress the instability.

Aims. In this paper, it is shown that the lower energy cutoff behavior of power-law electrons trapped in coronal loops can drive the ECM instability efficiently.

Methods. Based on the dispersive relation for high-frequency waves and distribution function for power-law electrons with a lower energy cutoff in a coronal loop, the growth rates of the $\text{O}$ and $\text{X}$ mode waves at fundamental and harmonic frequencies are calculated.

Results. The results show that the instability is driven when $\delta > \alpha$ because of a population inversion below the cutoff energy $E_\text{c}$, where $\delta$ is the steepness index describing the cut-off behavior and $\alpha$ the power-law spectrum index. The growth rates increase with $\delta$ and $E_\text{c}$, but decrease with $\alpha$, $\sigma$, and $\Omega$, where $\sigma$ is the magnetic mirror ratio of the loop and $\Omega$ the ratio frequency in the loop.

Conclusions. This novel driving mechanism for the ECM emission can be expected to have a potential importance for understanding the microphysics of radio bursts from the Sun and others.

Key words. masers – plasmas – radiation mechanisms: nonthermal – Sun: radio radiation

1. Introduction

Solar radio emission has attracted many authors’ attention during the past several decades. The key question is how a beam of fast electrons leads to the generation of electromagnetic waves. Ginzburg & Zhelezniakov (1958) are the pioneers in this field by advancing a theory to explain the observed fundamental (F) and harmonic (H) bands. Goldman (1983) also discussed this topic in subsequent years. Cairns (1987a–c), Robinson et al. (1993, 1994), Willes et al. (1996), Robinson & Cairns (1998a–c), Wu et al. (1994), and Yoon (1995, 1997, 1998) all put forward the plasma emission theory to explain the fundamental and harmonic emission in type III solar radio bursts. In this conventional theory, Langmuir waves that resulted from fast electrons play a pivotal role and are partly converted into electromagnetic waves by nonlinear wave-wave interaction. This model also incorporates large-angle scattering and reabsorption of fundamental emission amid ambient density fluctuations in the corona and solar wind. They all assume that the ambient magnetic field of the source regions of type III bursts is very weak. Although this approximation may be justified for source regions far enough away from the Sun, it is not obvious that it is appropriate for the emissions taking place near an active region in the low corona where nonthermal electrons are trapped by strong magnetic fields and are the main emitting sources of solar microwave bursts and spikes.

Another important theory was proposed by Wu & Lee (1979). In this model, radio emissions are produced by direct amplification of electromagnetic waves at the frequencies near the electron gyrofrequency and its harmonics. The ECM instability is the direct amplification mechanism for radio emissions in magnetized plasmas. With this theory, they can explain Earth’s auroral kilometric radiation well. In recent years, Wu et al. (2002, 2004, 2005), Chen et al. (2002), and Yoon et al. (2002) further developed this mechanism and applied it to explain type III solar radio bursts. In accordance with their model, amplified waves propagate in a magnetic flux tube until they arrive at a point where the frequencies of the excited waves are equal to the local exterior cutoff frequency. With some simplified models of magnetic field and electron density, some long-standing problems about type III solar radio bursts have been accounted for.

Most discussions of the ECM instability suppose that the nonthermal electrons have a loss-cone distribution, in which the perpendicular population inversion in electron energy is effective for driving the ECM instability (Chen et al. 2002). Observations from the hard X-ray, however, demonstrate that nonthermal flare-electrons approximately have a negative power-law distribution with a lower energy cutoff $E_\text{c}$ (Lin 1974; Gan et al. 2001), which will depress the growth rates of the instability. The cyclotron radiation from these power-law electrons can be regarded as the main source of microwave bursts in the GHz band (Kundu & Vlahos 1982; Aschwanden 2002; Stupp 2000; Fleishman 2004; Wu et al. 2007). In a recent work, Wu & Tang (2008) argue that the lower energy cutoff behavior of power-law electrons can drive the ECM instability efficiently even if the nonthermal electrons have an isotropic distribution.

In this paper, we apply this driving mechanism by the cutoff behavior to cases of coronal loops, where nonthermal electrons have an anisotropic distribution. The results show that the
growth rates of waves in the ordinary (O) and extraordinary (X) modes at the fundamental and harmonic frequencies increase with the steepness index $\delta$ of the cutoff behavior and the cutoff energy $E_c$ of the electron distribution function, but decrease with the power-law spectrum $\alpha$ of the distribution function and the magnetic-mirror ratio $\sigma$ and the ratio frequency (the electron gyrofrequency to the plasma frequency) $\Omega$ of the magneto-loop plasma.

This paper is organized as follows. In Sect. 2 we discuss the lower energy cutoff behavior of power-law electrons trapped in magneto-loops and introduce the distribution function. Then, the theory calculating the growth rate of the ECM instability is described in Sect. 3. The calculating results of the growth rates for the waves in the O and X modes at the fundamental and harmonic frequencies are discussed in Sect. 4. Finally, the summary and conclusions are presented in Sect. 5.

2. Lower energy cutoff of power-law electrons

According to a thick-target interaction mode (Brown 1971), the bremsstrahlung radiation of the power-law electrons caused by their interaction with the solar atmosphere also results in a power-law spectrum in the hard X-ray band, but with a different spectrum index. Thus, the electron spectrum index can be deduced from the observed hard X-ray spectrum. However, it is very difficult to determine a special form for the lower-energy cutoff behavior of a power-law electron event based on observations. Gan et al. (2001) discussed the two extreme cases of sharp cut-off spectrum index. Thus, the electron spectrum index can be described in terms of the magnetic-mirror ratio $\sigma$ and the ratio frequency (the electron gyrofrequency to the plasma frequency) $\Omega$ of the magneto-loop.

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of the plasma frequency \( \omega_p \), \( q = + \), and \( q = - \) denotes the O and X modes, respectively, and finally all frequencies have been normalized by \( \omega_p \).

For the \( n \)th harmonic with frequency \( \omega = n \Omega \), the growth rate of the ECM instability is given by (see e.g. Chen et al. 2002)

\[
\frac{\omega_{gr}}{\omega_{ce}} = \frac{\pi n_p}{2 n_0} \int d^3 u \frac{\gamma (1 - \mu^2)}{\omega_p (1 + T_q^2) R_q} \delta \left( \gamma - \frac{s \Omega}{\omega_p} - \frac{N_q \mu u}{c} \cos \theta \right) \times \left\{ \frac{\omega_p}{\omega} K_q \sin \theta + T_q \left( \gamma \cos \theta - \frac{N_q \mu u}{c} \right) J_b(\mu) \right\}
+ J_b'(\mu) \left( \frac{\partial}{\partial \mu} + \frac{N_q \mu \cos \theta}{c} - \mu \frac{\partial}{\partial \mu} \right) F_b(\mu, \sigma) \tag{6}
\]

where \( b_q = N_q \frac{\omega_p}{\Omega} \frac{u}{c} \sqrt{1 - \mu^2 \sin \theta} \), \( R_q = 1 - \frac{\Omega T_q}{2 \omega_p (\omega_p + T_q \Omega)} \left( 1 - \frac{q s_q}{\sqrt{s_q^2 + \cos^2 \theta \omega_p^2 - 1}} \right) \), \( K_q = -\frac{\Omega \sin \theta}{\omega_p \left( \omega_p^2 - 1 \right) (\omega_p + T_q \Omega)} \), \( T_q = -\frac{\cos \theta}{\tau_q} \).

The growth rate of the emission wave in the O and X modes can be calculated on the basis of the integral of Eq. (9) with the distribution \( F_b \) given by Eq. (2). For the given nonthermal electron parameters (\( \delta, \sigma \) and \( E_c \)) and the ambient magneto-plasma parameters (\( \Omega \) and \( \sigma \)) in the coronal loop, the growth rate depends on two variables \( (\omega_p, \theta) \). By peak growth rate, we mean the growth rate with highest magnitude as a function of one variable while the other is fixed. In contrast, the highest value in both \( (\omega_p, \theta) \) is called the maximum growth rate. We first discuss the dependence of the growth rate on the lower energy cutoff behavior, that is, on the steepness index \( \delta \) and the cutoff energy \( E_c \).

As an illustrative case, Fig. 2 shows the peak growth rates calculated by varying the frequency \( \omega_p \) for a given wave phase angle \( \theta \), where panels O1 and O2 are the fundamental \( (s = 1) \) and harmonic \( (s = 2) \) waves in the O mode, panels X1 and X2 are the fundamental and harmonic waves in the X mode, and different curves are for different steepness indices \( \delta = 3, 4, 5, 6, \) and 7 but a given spectrum index \( \alpha = 3 \) and cutoff energy \( E_c = 20 \text{ keV} \).

The ambient plasma parameters \( \Omega = 3.34 \) and \( \sigma = 10 \) have been used and the growth rate \( \omega \) normalized by \( \omega_{ce} n_p / n_0 \) in Fig. 2.

From Fig. 2, one can find that the growth rates are all negative for the saturation case of \( \delta \leq \alpha = 3 \) (the harmonic-wave growth rates for the O and X modes are negative, too, but not present in panels O2 and X2). For general steepness cases of \( \delta > \alpha \), the growth rates of the X mode waves \( (X_1 \text{ and } X_2) \) are considerably higher than those of the O mode waves \( (O_1 \text{ and } O_2) \). For the O mode, the growth rate of harmonic waves \( (O_2) \) is much less than that of fundamental waves \( (O_1) \) and is only \( -2\% \) of \( O_1 \), but in the X mode the growth rate of harmonic waves \( (X_2) \) is slightly lower than that of fundamental waves \( (X_1) \) by a factor \( \sim 1.5 \). In particular, it is worth noticing that these growth rates all increase with the steepness index \( \delta \). This implies that the steepness cutoff behavior with a positive slope (i.e., \( \partial \omega_p / \partial E > 0 \)) of power-law electrons indeed can efficiently excite the ECM instability. Moreover, the steeper cutoff behavior (i.e., with a larger steepness index \( \delta \)) more easily excites the ECM instability. For the case of the saturation cutoff of \( \delta \leq \alpha \), however, power-law electrons cannot excite the ECM instability because of the absence of population inversion.

In Fig. 2, it is also clearly shown that the growth rates in the four modes \( (O_1, O_2, X_1, \text{ and } X_2) \) all reach the maximum values at the direction close to perpendicular to the ambient magnetic field (i.e., \( \theta = \pi / 2 \)). Figure 3 plots the maximum growth rates as a function of the cutoff energy \( E_c \), where the parameters \( \alpha = 3, \delta = 4, \Omega = 3.34, \sigma = 10, \) and \( \theta = 1.58 \) have been used. From Fig. 3, it is clear that the growth rates all increase with the cutoff energy \( E_c \). It is found, again, that the growth rates of the X mode waves are considerably higher than those of the O mode waves and that the growth rate of \( \alpha = 3 \) is much higher than those of the


other three modes. It should be pointed out, however, that the X1 emission cannot escape from its astrophysical source, although it has the highest growth rate because its emitting frequency, $\omega_e = \Omega$, is always below the cutoff frequency for the X mode, $\Omega_{Xc} = \sqrt{\Omega^2/4 + 1 + \Omega/2}$. In consequence, the dominating modes are in reality the X2 and O1 emissions, which have growth rates very close for a wide range of $E_c$.

Next we consider the dependence of the growth rate on the spectrum index $\alpha$ of power-law electrons. Figure 4 plots the peak growth rates versus the wave phase angle $\theta$, where panels are denoted in the same way as in Fig. 2, but different curves are for different spectrum indices $\alpha = 2, 3, 4, 5$, and 6 and a given steepness index $\delta = 6$ and cutoff energy $E_c = 20$ keV. The ambient plasma parameters $\Omega = 3.34$ and $\sigma = 10$ have been used. From Fig. 4, one can find, again, that the growth rates all are negative for the saturation case of $\alpha = \delta = 6$. The harmonic-wave growth rates for the O and X modes are negative, too, but not present in the panels O2 and X2. For general steepness cases of $\alpha < \delta$, the growth rates all decrease with the spectrum index $\alpha$. This indicates that the negative power-law spectrum of energetic electrons will depress the ECM instability and that a softer spectrum (i.e., with a larger spectrum index $\alpha$) leads to a lower growth rate.

Finally, we discuss the dependence of the growth rate on the ambient plasma parameters, the ratio-frequency $\Omega = \omega_{pe}/\omega_p$, and the magnetic mirror-ratio $\sigma = B_{max}/B_{min}$. Figure 5 presents the peak growth rates varying with the wave phase angle $\theta$, where panels are denoted in the same way as above, and different curves are for different ratio-frequencies $\Omega = 2, 3.34, 5$, and 10 but given other parameters $\alpha = 3, \delta = 4, E_c = 20$ keV, and $\sigma = 10$. The result shows that, with $\Omega$ decreasing, the growth rates become higher and the unstable range of $\theta$ wider except for the X1 mode that cannot escape from its astrophysical source region. In Fig. 5, the curve with $\Omega = 10$ for the O2 mode does not present in the panel O2 because its growth rate is too low.

Figure 6 presents the dependence of the mirror-ratio $\sigma$ in the coronal loop on the growth rates, where the parameters $\alpha = 3, \delta = 4, E_c = 20$ keV, $\Omega = 3.34$, and $\theta = 1.58$ have been used. From Fig. 6, one can find that the growth rates all decrease with $\sigma$. In particular, the growth rates are sensitive to the mirror ratio for small mirror ratios of $\sigma < 5$ and decrease rapidly with $\sigma$, but for high mirror-ratios of $\sigma > 10$ the growth rates approach constants. By the way, as denoted in the curve of O2, the growth rate of the O2 mode has been enlarged by a factor of 50 because of how low it is.

5. Summary and conclusions

The ECM instability, which can be driven by nonthermal electrons trapped in magnetic fields, is an important radiation mechanism in astrophysics and has been extensively applied to various short-time radio-burst phenomena, such as the auroral kilometric radiation from the Earth, radio emission from other magnetized planets (e.g. Jupiter and Saturn) in the solar system, and the kilometric continuum radiation from the wind of other stars. In fact, the most interesting of these considerations are the results for the harmonic waves in the X mode and the O mode, which are deviated from the results for the fundamental waves in the X mode and the O mode.

**Fig. 2.** Peak growth rates driven by the ECM instability: O1, fundamental waves in the O mode; O2, harmonic waves in the O mode; X1, fundamental waves in the X mode; X2, harmonic waves in the X mode.

**Fig. 3.** Maximum growth rates versus the cutoff energy $E_c$: O1, fundamental waves in the O mode; O2, harmonic waves in the O mode; X1, fundamental waves in the X mode; X2, harmonic waves in the X mode.
system and extrasolar planets, radio bursts or spikes from the Sun and other stars, and the time-varying emission from blazar jets (see Treumann 2006, for a recent review). Astrophysical observations, on the other hand, demonstrate that nonthermal electrons frequently present in a power-law distribution with a lower-energy cutoff. In this paper, we investigated the growth rates of the ECM emissions of fundamental and harmonic waves in the O and X modes driven by power-law electrons trapped in coronal magneto-loops. The trapped power-law electrons may have a loss-cone distribution due to the magnetic mirror-force effect of the loop magnetic field on the power-law electrons. We introduced a new distribution function of power-law electrons that describes not only a continual and smooth loss-cone boundary but also a continual and smooth lower energy cutoff behavior. Based on the weakly relativistic approximation, we discussed the dependence of the growth rates on the parameters of both the power-law electrons and the ambient magneto-plasma in the coronal loop.

Results from our calculations show that the power-law electrons with the steepness cutoff (i.e., \( \delta > \alpha \)) can excite the ECM instability efficiently because of the energy reverse distribution just below the cutoff energy \( E_c \), and the growth rates of
fundamental and harmonic waves in both the O and X modes all increase with both the steepness index $\delta$ and the cutoff energy $E_c$. Also the results show that the $X_2$ and $O_1$ mode waves dominate the radiation driven by the ECM instability. On the other hand, for the saturation cutoff case of $\delta \leq \alpha$, the power-law electrons cannot drive the ECM instability because of the absence of population inversion.

We also discussed the dependence of the growth rate on the spectrum index $\alpha$. The result indicates that the negative power-law spectrum of nonthermal electrons will depress the growth rate of the ECM instability and that a softer spectrum (i.e., with a larger spectrum index $\alpha$) leads to a lower growth rate.

Finally, the growth rate of the ECM instability sensitively depends on the ambient magneto-plasma parameters too, the ratio-frequency $\Omega = \omega_{ce}/\omega_p$ and the magnetic mirror-ratio $\sigma = B_{\text{max}}/B_{\text{min}}$ in the coronal loop. The result shows that, with $\Omega$ decreasing, the growth rates become larger and the unstable range of $\theta$ wider except for the $X_1$ mode that cannot escape from its astrophysical source region. On the other hand, the growth rates decrease rapidly with $\sigma$ for low mirror ratios of $\sigma < 5$, but approach constants for high mirror ratios of $\sigma > 10$.

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