

## Erratum

# Second-order torque on the tidal redistribution and the Earth's rotation

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### ABSTRACT

The numerical results presented in our paper under the same title (Lambert & Mathews 2006, A&A, 453, 363), have been found to be vitiated by certain errors mentioned below. The results of new calculations now done are tabulated and the main changes from the earlier results are pointed out. Specially notable is the obliquity rate, which is now found to be 1.84 mas/cy, within the range of values found in the earlier literature.

**Key words.** Earth – reference systems – errata, addenda

While investigating the reason for the discrepancy of an order of magnitude between the result shown in our earlier paper (Lambert & Mathews 2006, hereafter referred to as LM06) for the obliquity rate contribution from the action of the tidal potential on the tidal redistribution of matter in the Earth and the results obtained by different means by other workers, we discovered certain errors in our computation of the incremental inertia tensor of the Earth due to ocean tidal (OT) effects. These errors mean that the numerical values presented in LM06 for the second-order contributions to the nutation as well as to the precession in longitude and to the obliquity rate, are incorrect. Although the new results do not contradict the main conclusions of the paper, it is necessary that we provide the corrected values, text and tables.

Two mistakes occurred in using Eq. (42) of LM06 for the numerical computation of the ocean tidal increments to the geopotential coefficients. Firstly, the expression (43) for the quantity  $L_{lm}$  is incorrect: the factor  $K_{lm}$  (which relates the “barred” and “unbarred” geopotential coefficients) should not be there. Equation (44) is then irrelevant. Secondly, an error in the process of conversion of the Doodson arguments of spectral components of ocean tides to the corresponding Delaunay arguments that are appropriate for nutation components, resulted in a reversal of the sign of the argument  $\Theta_s$  in (42). These errors propagated into the numerical computations of the spectral components of the OT contributions to the second order torques  $\Delta\Gamma_{2m}$ .

Finally, it was a mistake to apply a rotation of 14°95 West, as stated in the paragraph following Eq. (47), to rotate the OT increments of inertia from the terrestrial frame having its  $x$ -axis along the prime meridian into the frame defined by the principal axes of inertia. No such rotation was necessary, as we had taken the

time dependence of  $\tilde{\phi}$  to be through the same factor  $e^{-i\Theta}$ . This is similar to the OT increments to the geopotential (Eq. (42)) and to the inertia tensor (Eqs. (45, 46)). It is to be remembered that triaxiality has a negligible effect on the tidal deformations that we have been considering. Therefore the following three sentences in the above-mentioned paragraph have to be dropped: “note that the tabulated OT data are with reference to the terrestrial frame with its  $x$  axis in the Greenwich meridian, while our dynamical equations are expressed in the frame defined by the principal axes of inertia. The latter frame is rotated to the West by 14°95 relative to the former (see Bretagnon et al. 1997). We have taken account of this difference between the two frames in making our calculations relating to the sectorial tides”.

We have now recalculated the values of the second order contributions to the nutation, precession in longitude, and obliquity rate, avoiding the above-mentioned mistakes. Tables 3 and 3 of the present Erratum display the corrected values. (Since columns for the elastic case only, tagged by EL in LM06, remain the same, they are not reproduced in the tables hereafter.) The main feature of Table 3, namely the almost complete cancellations changes between the contributions from reciprocal actions (between the first and second, as well as between the third and fourth sections of the table), remains unchanged. The sum total of the contributions is now  $-39 \mu\text{as}$  to the in-phase and  $-29 \mu\text{as}$  to the out-of-phase 18.6-yr nutation in longitude (instead of  $-35$  and  $-4$  as earlier), and there is now also an out-of-phase contribution of  $-10 \mu\text{as}$  to the 18.6-yr nutation in obliquity. The most important change is in the obliquity rate: our revised value is 1.84 mas/cy (replacing the LM06 value of 0.13 mas/cy). It is close to the literature values, namely, 2.44 mas/cy of Williams (1994) who obtained it by applying the concept of conservation

**Table 2.** Contributions to nutations (in  $\mu\text{as}$ ) due to the the interaction of tidal potentials and deformations of different orders. Boxes with values below  $1 \mu\text{as}$  are left blank. EL: Elastic Earth, AE: Anelasticity, OT: Ocean Tides.

$l$	$l'$	$F$	$D$	$\Omega$	Period (days)	EL+AE+OT			
						$\Delta\psi$		$\Delta\epsilon$	
						sin	cos	sin	cos
Zonal potential on Tesserall tides									
0	0	0	0	1	-6798.4	168	-2	-5	-8
0	0	0	0	2	-3399.2	5			-3
0	0	2	-2	2	182.6	4	-1		
Tesserall potential on Zonal tides									
0	0	0	0	1	-6798.4	-214	-2		11
0	0	0	0	2	-3399.2	-6			3
0	0	2	-2	2	182.6	-5			
0	0	2	-2	3	187.7	-1			
Tesserall potential on Sectorial tides									
0	0	0	0	1	-6798.4	58	-14	-3	-57
0	0	2	-2	2	182.6	16	-1		-8
0	0	0	0	2	-3399.2	-3			2
0	0	2	0	2	13.7	2			-1
0	0	0	2	0	14.8	-2			
0	0	2	-2	1	177.8	2			-1
1	0	0	0	0	27.6	1			
Sectorial potential on Tesserall tides									
0	0	0	0	1	-6798.4	-51	-12	-2	55
0	0	2	-2	2	182.6	-17	-1	-1	8
0	0	0	0	2	-3399.2	4			-2
0	0	2	0	2	13.7	-3			1
0	0	0	2	0	14.8	2			
0	0	2	-2	1	177.8	-2			1
1	0	0	0	0	27.6				
Total									
0	0	0	0	1	-6798.4	-39	-29	-10	1
0	0	2	-2	2	182.6	-3	-3	-1	1
0	0	2	-2	3	187.7	-1			

of the sum of the angular momenta of rotation of the Earth and of the orbital motion of the Moon, and the result of Kaula (1964) which is  $\approx 1.7 \text{ mas/cy}$  after adjustment by Williams for the latest values of the parameters involved.

**Table 3.** Contributions to the precession in longitude and the obliquity rate (in  $\mu\text{as/cy}$ ) due to the interactions between different types of tidal potentials and deformations. Boxes with values below  $1 \mu\text{as/cy}$  are left blank. EL: Elastic Earth, AE: Anelasticity, OT: Ocean Tides.

Potential	Tides	EL+AE+OT	
		$\psi$	$\epsilon$
Zonal	Tesserall	-3657	121
Tesserall	Zonal	5520	14
Tesserall	Sectorial	-25 287	903
Sectorial	Tesserall	23 300	806
Total		-124	1844

As a check of the reliability of the LM06 theoretical framework and of the calculation of the OT increment of inertia, we have calculated the third component of the second order torque, and hence the rate of secular increase in the length of day, by a direct extension (to be presented elsewhere) of this theoretical framework; we find the rate due to ocean tidal effects to be  $2.34 \text{ ms/cy}$ , close to the rate of  $2.28 \text{ ms/cy}$  obtained by Ray et al. (1999).

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