

# The statistical significance of the superhump signal in U Geminorum

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## ABSTRACT

**Context.** Although its well-determined mass ratio of  $q = M_{\text{sec}}/M_{\text{wd}} = 0.357 \pm 0.007$  should avoid superoutbursts according to the thermal tidal instability model, in 1985 the prototypical dwarf nova U Gem experienced an extraordinary long outburst very much resembling superoutbursts observed in SU UMa systems. Recently, the situation for the model became even worse as superhump detections have been reported for the 1985 outburst of U Gem.

**Aims.** The superhump signal is noisy and the evidence provided by simple periodograms seems to be weak. Therefore and because of the importance for our understanding of superoutbursts and superhumps, we determine the statistical significance of the recently published detection of superhumps in the AAVSO light curve of the famous long 1985 outburst of U Gem.

**Methods.** Using Lomb-Scargle periodograms, analysis of variance (AoV), and Monte-Carlo methods we analyse the 160 visual magnitudes obtained by the AAVSO during the outburst and relate our analysis to previous superhump detections.

**Results.** The 160 data points of the outburst alone do not contain a statistically significant period. However, using the characteristics of superhumps detected previously in other SU UMa systems additionally and searching only for signals that are consistent with these, we derive a  $2\sigma$  significance for the superhump signal. The alleged appearance of an additional superhump at the end of the outbursts appears to be statistically insignificant.

**Conclusions.** Although of weak statistical significance, the superhump signal of the long 1985 outburst of U Gem can be interpreted as further indication of the SU UMa nature of this outburst. This further contradicts the tidal instability model as the explanation of the superhump phenomenon.

**Key words.** accretion, accretion disks – instabilities – stars: individual: U Geminorum – stars: dwarf novae, cataclysmic variables – stars: binaries: close

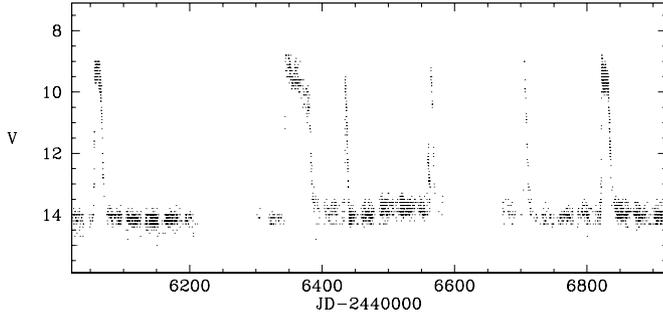
## 1. Introduction

Dwarf novae are non-magnetic CVs showing quasi-regular outbursts, i.e., increased visual brightness of 2–5 mag for several days, which typically reappear on timescales of weeks to months (e.g. Warner 1995, for a review). SU UMa stars are short-period, i.e.,  $P_{\text{orb}} \leq 2.2$  h, dwarf novae whose light curves consist of two types of outbursts: normal dwarf nova outbursts and superoutbursts, which are 5–10 times longer, as well as  $\sim 0.7$  mag brighter. These superoutbursts also show pronounced humps (called superhumps) reappearing with periods usually a few percent longer than the orbital one. The phenomenon is usually explained by tidal disc deformations when the radius of the disc reaches the 3:1 resonance radius (Whitehurst 1988; Whitehurst & King 1991; Lubow 1991). This resonance is possible only if the mass ratio of the components is small, i.e.  $q \equiv M_{\text{sec}}/M_{\text{wd}} \leq 0.33$ . Therefore the observed appearance of superhumps in systems with short orbital periods and small mass ratios is in general agreement with the tidal instability explanation of *superhumps*. In contrast, there have been two possible scenarios for the triggering mechanism of *superoutbursts* proposed: either they are also caused by the 3:1 resonance as it is assumed in the thermal tidal instability (TTI) model (see, e.g., Osaki 1996, for a review) or they are triggered by enhanced mass transfer (EMT) as proposed by Vogt (1983) and Smak (1984). According to the disc instability model, the EMT scenario appears to be more plausible

(Schreiber et al. 2004). On the other hand, the mechanism claimed to cause the mass transfer enhancement, i.e. irradiation of the secondary by the white dwarf and the boundary layer is not well understood (see Osaki & Meyer 2003; Smak 2004b; Osaki & Meyer 2004; Smak 2004a; Schreiber et al. 2004; Truss 2005, for recent arguments on the EMT and the TTI models).

The prototypical dwarf nova system U Gem plays a key role in the context of the discussion about superoutbursts, superhumps, the EMT, and the TTI model: Its orbital period is 4.25 h and its mass ratio of  $q = 0.357 \pm 0.007$  (Naylor et al. 2005) is above the limit for tidal instabilities. U Gem normally shows regular dwarf nova outbursts with 10–20 days duration and recurrence times of  $\sim 100$  days typical for long orbital period dwarf novae. In 1985 U Gem exhibited a famous long (45 days) and large amplitude ( $\sim 0.5$  mag brighter) outburst reminiscent of a SU UMa superoutburst (see Fig. 1 or Mason et al. 1988). According to the tidal instability model, the mass ratio of U Gem should prevent the appearance of superoutbursts. Therefore, the 1985 outburst may indicate that superoutbursts are not caused by tidal instabilities (e.g., Lasota 2001).

Recently the situation became even more difficult for the TTI model: Smak & Waagen (2004) reported the detection of superhumps in the AAVSO light curve of the 1985 outburst of U Gem which – if true – has extremely far-reaching consequences. Superhumps in U Gem would (1.) contradict the general explanation of the superhump phenomenon and

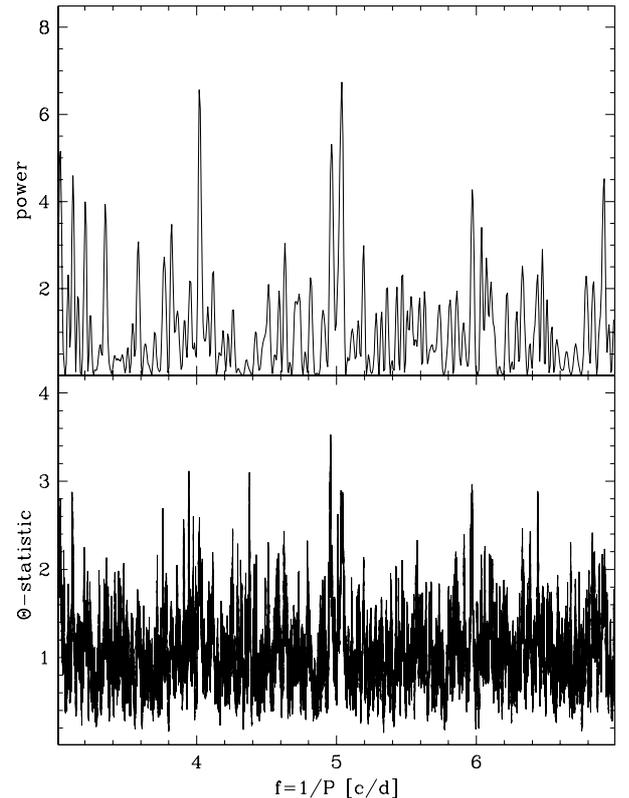


**Fig. 1.** Snapshot of the visual light curve of U Gem including the extremely long “superoutburst” (starting at JD  $\sim$  2 446 344) for which Smak & Waagen (2004) reported the detection of superhumps. The data has been provided by the AAVSO.

(2.) synchronise the simultaneous appearance of superhumps and superoutbursts. The detection of superhumps in U Gem would hence require developing a new theory for superhumps and superoutbursts that should not rely on tidal forces (see also Hameury & Lasota 2005a,b). Such a new scenario would be in agreement with the findings of Kornet & Rozyczka (2000), whose hydrodynamic TTI models do not predict superhumps if the full energy equation is taken into account. However, before completely abandoning the thermal-tidal instability model, one should be aware that determining periods in uneven datasets is a non-trivial statistical exercise. In particular, it is difficult to estimate the significance of a signal in a periodogram. Smak & Waagen (2004) list many arguments why they *believe* in the reality of the superhump periodicity ranging from the coherence of the period changes (their Fig. 4) to the fact that the amplitude of the alleged superhump signal is typical for this phenomenon. However, the signal shown in the periodograms is extremely weak and has been taken with some scepticism (Patterson et al. 2005). The claimed detection of superhumps in U Gem is of outstanding importance for both our understanding of the superhump phenomenon itself, as well as the triggering mechanism of superoutbursts. Therefore we want to qualify the *belief* and the *scepticism* and present here a detailed analysis of the AAVSO data using not only discrete Fourier transforms, but also analysis of variances (AoV) and randomisation techniques to determine the statistical significance of the alleged periodicities.

## 2. Periodograms

Having selected the 160 AAVSO measurements representing the plateau of the long 1985 outburst of U Gem (JD = 2 446 344.66–2 446 381.43), we first analyse the data following Smak & Waagen (2004): we subtract the linear trend and calculate a simple Lomb-Scargle periodogram according to a classical discrete Fourier transform to construct a power spectrum. In addition, it is common and useful to analyse uneven time series of data using the method of phase dispersion minimisation (PDM) (Stellingwerf 1978) or AoV (Schwarzenberg-Czerny 1989). Figure 2 shows periodograms obtained with the Lomb-Scargle algorithm and the AoV method. Indeed, there is a peak at a frequency  $f \sim 5$  c/d in the periodograms as reported by Smak & Waagen (2004), and the double-peak shape indicates that the period could vary with time. We here repeat the question asked by these authors: “Does this signal represent a real periodicity?” Unfortunately, answering this question, i.e., determining the statistical significance of period detections in uneven datasets is not straightforward. In fact, a perfect analytical solution of the



**Fig. 2.** Periodograms for the AAVSO data of the long and bright 1985 outburst of U Gem using the Lomb-Scargle algorithm and the AoV method. The power-spectrum and the analysis of variance (AoV) statistics are calculated for  $1 \times 10^5$  frequencies. The binning in the case of the AoV periodogram is  $N_c = 2$ ,  $N_b = 10$ . The double-peaked shape of the claimed superhump signal around  $f \sim 5$  c/d indicates that the period may be variable in time.

problem does not yet exist. To determine the significance for time-dependent signals and/or one particular time spacing of the data points, one can either use numerical simulations or semi-analytical approximations.

## 3. Significance tests

Using the Lomb-Scargle formalism one can determine the significance using estimates for the number of independent frequencies  $M$ . The false alarm probability is then given by

$$p(>z) \equiv 1 - (1 - e^{-z})^M, \quad (1)$$

where  $z$  is the normalised power of the most significant peak in the periodogram (see Press et al. 1992). In general  $M$  depends on the frequency bandwidth, the number of data points, and their detailed time spacing. While Horne & Baliunas (1986) performed extensive Monte-Carlo simulations to determine  $M$ , Press et al. (1992) argue that one does not need to know  $M$  very precisely and that it usually should not be essentially different from the number of data points  $N_0$ . More recently, Paltani (2004, his Sect. 3.2) presented an interesting and new semi-analytical method to estimate  $M$ . As this formalism requires less extensive numerical simulations than the Horne & Baliunas (1986) approach, it certainly represents a promising new method.

Alternatives to the Lomb-Scargle method are PDM and AoV. In both cases we know the probability distribution of the statistic  $\Theta(P)$  if the period ( $P$ ) is known (i.e., the beta and the

Fisher-Snedecor (F) distribution Schwarzenberg-Czerny 1997, 1989). Unfortunately, replacing  $e^{-z}$  in Eq. (1) with the corresponding beta or F probabilities leads to similar problems to above, i.e., one needs to estimate the number of independent frequencies  $M$  (see also Heck et al. 1985). Instead Linnell Nemeč & Nemeč (1985) proposed a Fisher randomisation technique to numerically determine the required probability distribution of the maximum of the statistics in the considered period range  $V$ , i.e.,  $\max(\Theta(P))$  for  $P \in V$ . Assuming that there is no periodicity in the data, the observations should be independent of the observing times and randomly redistributing the measurements should not give significantly different results. Generally speaking, the null hypothesis, i.e., assuming that the variations in the data represent just noise, is true if the original periodogram does not contain a particularly strong peak when compared to those obtained after having randomly redistributed the observations. More specifically the method is as follows. First one calculates periodograms and the maximum of the statistics ( $\max(\Theta(P))$ ) for the observations using the PDM or AoV technique. Then, this procedure is repeated many times after having randomly redistributed the measurements over the fixed times of observations each time. This Monte-Carlo experiment results in a distribution of maxima of the  $\Theta$ -statistic. The proportion of randomised datasets producing a value of  $\max(\Theta(P))$  that is equal to or larger than the value obtained for the observations gives the probability value  $p$  (often also called the probability of error). The statistical significance of the detected signal is  $1 - p$ . The error of  $p$  obviously depends on the number of randomised datasets  $n$ , on the resolution in frequency, and the binning. Following Linnell Nemeč & Nemeč (1985) the standard error of  $p$  can be approximated as  $\sigma_p \sim [p(1-p)/n]^{1/2}$  and the 95% confidence level can be written as  $p \pm 2\sigma_p$ .

In the following sections we use AoV and the just-described randomisation method to determine the statistical significance of the superhump signal in the light curve of the 1985 outburst of U Gem. To assure that our results are independent on the resolution we increased the number of frequencies until the obtained value of  $p$  remained constant. We find that calculating  $\Theta(P)$  for  $10^5$  equally spaced frequencies in the range of  $3 \leq f \leq 7$  is sufficient to exclude artefacts resulting from low resolutions. In addition, we performed several randomisation tests to estimate the influence of different binnings. The binning parameters we used are  $N_b$  and  $N_c$  as defined in Stellingwerf (1978). It turns out that  $p$  somewhat depends on the binning as long as  $N_c = 1$ , but remains more or less constant for  $N_c \geq 2$ . Finally, we want to stress that although often very useful, Monte-Carlo or randomisation techniques have their limitations. Most importantly, they rely on the assumption of white noise, i.e., that individual observations are not correlated. In contrast, true observations may contain a degree of correlation and this effect can interfere with periodogram statistics. To assure that the conclusions of this article are not affected by the limitations of randomisation techniques, we compare our final results with semi-analytical estimates following Press et al. (1992) and Paltani (2004, his Sect. 3.2).

#### 4. The superhump signal

To analyse the observations, we first subtract the linear trend and the offset from the visual magnitudes to get a distribution of 160 values of  $\Delta V$  with the mean value at zero as Smak & Waagen (2004) did. One can then analyse the observed light curve using the AoV method and the described randomisation techniques to determine the significance of the claimed

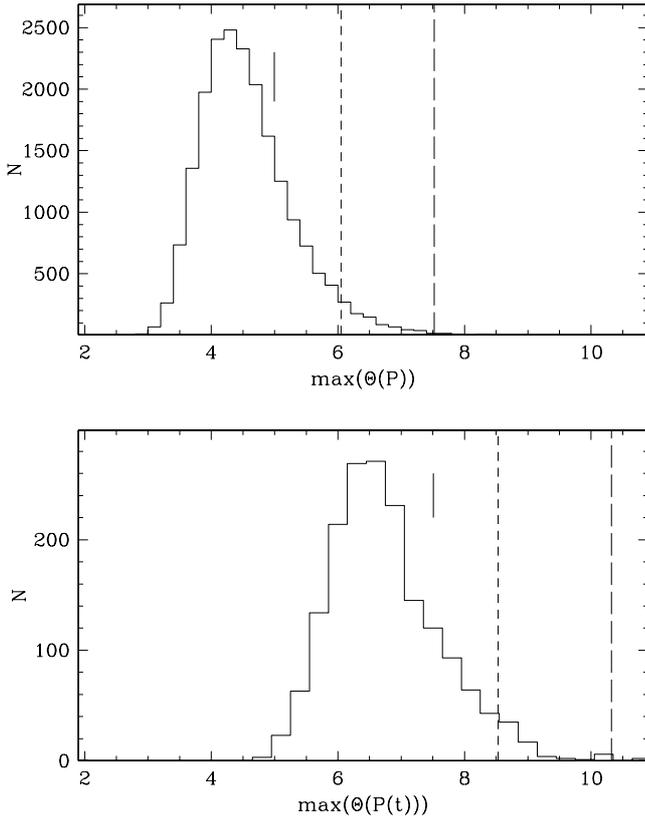
superhump signal. However, simply analysing periodograms with constant trial periods is not sufficient in the context of superhumps as their periods usually variable in time. Another important boundary condition for the numerical method is given by the considered range of periods. Using additional information, e.g., derived from earlier observations of superhumps, may significantly affect the results. For example, if one can restrict the range of trial periods, a signal that has not been significant may become significant under the condition given by the additional information. To analyse the statistical significance of the alleged superhump signal, we discuss pure periodogram analysis, time dependence, and the restriction to a small range of trial periods in the following three sections.

##### 4.1. Pure periodogram analysis

Using a rather broad range of constant trial periods corresponding to  $7c/d \leq f \leq 3c/d$ ,  $N_b = 5$ ,  $N_c = 2$ , and  $2 \times 10^4$  randomised datasets, we find that the alleged superhump signal is not statistically significant, i.e.,  $p = 0.275 \pm 0.003$ . This means that one finds a signal of the same significance in more than one-fourth of all light curves. This result exactly reflects the impression mentioned by Patterson et al. (2005) that the evidence for the superhump signal does not seem strong.

##### 4.2. Time dependence

In the previous section we have shown that the AAVSO data does not contain a significant *constant* period. This does, however, *not* answer the question whether the superhump detection by Smak & Waagen (2004) is real or not because we used only constant trial periods, i.e., we ignored that superhump periods in general and the one claimed for U Gem in particular are time dependent. Smak & Waagen (2004) find that period changes of the alleged superhump period are coherent and give a final fit for the times of maxima (their Eq. (3)). We calculate the value of the AoV statistic for this time-dependent period and obtain  $\Theta(P(t)) = 7.51$ . Comparing this with the previously obtained constant value  $\Theta(P) = 5.00$  shows that indeed the time-dependence of the alleged superhump period increases the value of the statistic. However, one has to consider that the probability distribution changes if one takes the time-dependence of the period into account. To determine the significance we need to compare the signal with the distribution of  $\max(\Theta(P(t)))$  where  $P(t)$  represents every type of time-dependence acceptable for superhumps. We have incorporated this in our Monte-Carlo simulations by using additional time-dependent trial periods restricting ourself to coherent period changes. We used  $P(t) = P_0 + a\dot{P}$ , where  $1/3d \geq P_0 \geq 1/7d$  and  $-0.0001 \leq a \leq 0.0001$ . This is certainly reasonable as most superhump period derivatives and especially the one measured for U Gem are constant. The effect of taking time dependence into account is illustrated in Fig. 4. On the left-hand side we used a broad range of trial periods and the alleged superhump signal is hardly distinguishable from noise. In other words, allowing for  $dP/dt \neq 0$  does not significantly increase the statistical significance. The bottom panel of Fig. 3 shows the corresponding distribution of  $\max(\Theta(P(t)))$  derived from our high resolution Monte-Carlo simulations. The vertical lines indicate the  $\Theta$ -statistic required for  $2\sigma$  and  $3\sigma$  significance. Obviously, even the time-dependent signal does not reach these values.



**Fig. 3.** Distribution of the maximum of the  $\Theta$ -statistic for 5 bins and using a broad range of trial periods, i.e.,  $3 \text{ c/d} \leq f \leq 7 \text{ c/d}$ . In the top panel we used only constant periods, while we took into account coherent time evolution for the superhump periods in the bottom panel. The short solid vertical lines indicate the positions of the claimed superhump signal. The dashed lines represent the 95% and 99.7% ( $3\sigma$ ) significance levels. In both cases the statistical significance of the superhump signal is far from reaching  $2\sigma$ , i.e.,  $p = 0.275 \pm 0.003$  (top panel) and  $p = 0.224 \pm 0.010$  (bottom panel).

#### 4.3. The range of periods

It has been realised by Smak & Waagen (2004) that the periodogram alone cannot confirm the appearance of superhumps in U Gem. Therefore they argue that the superhump period and the corresponding value of  $\epsilon \equiv (P - P_{\text{orb}})/P_{\text{orb}} = 0.13$  are consistent with the mass ratio of U Gem and its long orbital period. This led the authors to the reasonable interpretation that the signal represents a real superhump. Here, we test this argument using the numerical methods described above.

Figure 5 shows the  $\epsilon - P_{\text{orb}}$  relation for SU UMa stars and the position of U Gem. Indeed, the claimed superhump signal lies directly on the extension of the linear relation for SU UMa systems. We approximated the  $\epsilon - P_{\text{orb}}$  relation using linear regression (Fig. 5). The regions defined by the 1, 2, 3 $\sigma$  errors of the linear fit are also shown. Assuming that we know with 100% confidence that the  $\epsilon - P_{\text{orb}}$  relation has a linear extension up to long orbital period dwarf novae like U Gem, a new superhump signal should lie in the 3 -  $\sigma$  region around the linear fit with a probability of 0.997. Using this information means analysing only those periods that are in agreement with the extension of the  $\epsilon - P_{\text{orb}}$  relation, i.e.,  $4.83 \text{ c/d} \leq f \leq 5.24 \text{ c/d}$ . In the case of time-dependent periods, the effect is illustrated in Fig. 4. While the superhump signal does not produce a significant peak in the  $f$ - $dP/dt$  plane if a broad range of trial periods is used (left), it is clearly the highest peak in case the range of trial

periods is restricted to periods in agreement with the  $\epsilon - P_{\text{orb}}$  relation (right). This restriction indeed dramatically increases the significance of the signal as the distributions of  $\max(\Theta(P))$  and  $\max(\Theta(P(t)))$  are shifted towards smaller values. The results of our detailed Monte-Carlo simulations are shown in Fig. 6. The  $p$ -value is below the  $2\sigma$  significance level in both cases, i.e.,  $p = 0.0381 \pm 0.001$  in the case of constant trial periods (top panel) and  $p = 0.0384 \pm 0.003$  when taking into account coherent period changes (bottom panel).

#### 4.4. Semi-analytical methods

As mentioned in Sect. 2, Monte-Carlo methods may fail if the analysed observations contain correlated data. To make sure that our results are not affected by this effect, we use semi-analytical methods to estimate the number of independent frequencies  $M$ . The power of the signal in the Lomb-Scargle periodogram (Fig. 2) and Eq. (1) then allow us to derive a statistical significance which can be compared with the results obtained by our extended numerical simulations.

Following Press et al. (1992),  $M$  should not be very different from the number of data points, i.e.,  $M \sim N_0 = 160$ . This estimate for  $M$  is, however, valid for the frequency range  $0 \leq f \leq f_N$  with  $f_N$  being the Nyquist frequency. Assuming that  $M$  decreases linearly with the frequency bandwidth (see Press et al. 1992, for a discussion), the restriction to the small range of frequencies, i.e.,  $4.83 \text{ c/d} \leq f \leq 5.24 \text{ c/d}$ , leads to  $M \sim 30$ . Using Eq. (1) and  $z = 6.74$  we derive

$$M \simeq 30 \rightarrow p \simeq 0.035. \quad (2)$$

This is obviously in perfect agreement with our detailed Monte-Carlo simulations.

As an additional final (and less rough) test, the Linnell Nemeč & Nemeč (1985) method and the Press et al. (1992) estimate can be compared with the formalism recently proposed by Paltani (2004, his Sect. 3.2). For  $n$  frequencies and an arbitrary threshold  $\Theta^*$ , this method requires  $n \text{Prob}(\Theta \geq \Theta^*) \ll 1$ . We follow Paltani in using  $n \text{Prob}(\Theta \geq \Theta^*) = 0.1$ . With  $n = 1000$  frequencies in the range of  $4.83 \text{ c/d} \leq f \leq 5.24 \text{ c/d}$ , this requires  $\text{Prob}(\Theta \geq \Theta^*) = 0.0001$ , which (according to the Fisher-Snedecor distribution for AoV statistics) results in  $\Theta^* = 6.3$ . For  $m = 10.000$  randomised data sets we then determine the maximum value of  $\Theta(f_{j=1, \dots, n})$  and the number of data sets with  $\max(\Theta(f_j)) > \Theta^*$  gives an estimate for the number of independent frequencies  $M$ , i.e.,

$$M = \#(\max_{(j=1, \dots, n)} \Theta(f_j) > \Theta^*). \quad (3)$$

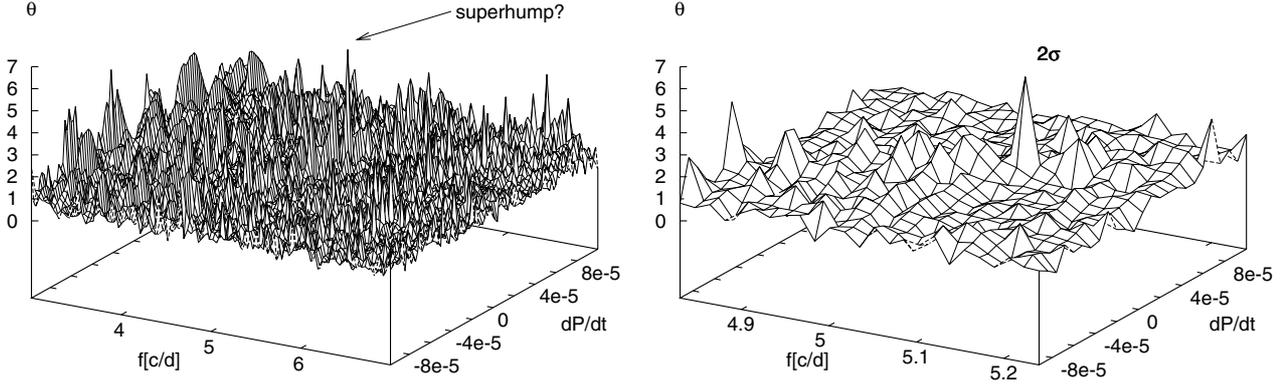
Approximating  $\#(\max_{(j=1, \dots, n)} \Theta(f_j) > \Theta^*)$  with a Poisson distribution, the error of  $M$  is simply  $\Delta M = \sqrt{M}$  and we finally obtain

$$M = 28 \pm 5 \rightarrow p = 0.033 \pm 0.006. \quad (4)$$

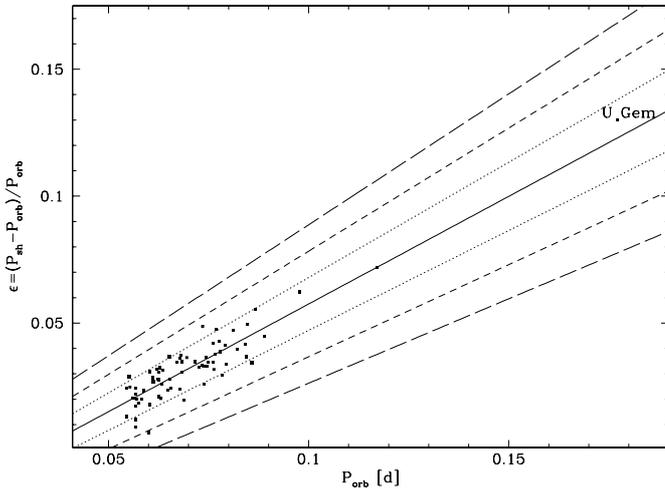
This is again in very good agreement with our previous results.

## 5. The additional superhump

In addition to the superhump signal at  $f \sim 5 \text{ c/d}$ , Smak & Waagen (2004) claimed the existence of an additional superhump around  $f \sim 5.55$  appearing during the final stages of the outburst. To determine the significance of this signal we analysed separately the last 40 data-points (covering JD = 2 446 366–22 122 446 382). Indeed, the strongest signal is no



**Fig. 4.** Low resolution AoV periodograms taking into account a range of constant period derivatives illustrating the effect of restricting the range of trial periods. The periodograms are calculated using  $N_b = 5$  and  $N_c = 2$ . We restricted our analysis to constant period derivatives, i.e., we used  $P(t) = P_0 + a\dot{P}$  where  $-0.0001 \leq a \leq 0.0001$ . The left panel shows the periodogram for a broad range of trial periods i.e.,  $1/3 \text{ d} \geq P_0 \geq 1/7 \text{ d}$ . The peak at  $f \sim 5 \text{ c/d}$  and  $dP/dt \sim 2e-5$  is hardly distinguishable from the noise and is not statistically significant. However, if we search only for periods consistent with the  $\epsilon - P_{\text{orb}}$  relation, i.e.,  $4.83 \text{ c/d} \leq f \leq 5.24 \text{ c/d}$  (see Fig. 5), the peak produced by the superhump signal is clearly the highest peak and we derive a  $2\sigma$  significance. Please note, the data used to construct the figures above has been calculated with a much lower resolution in  $f$  and  $dP/dt$  than used in our detailed Monte-Carlo simulations.

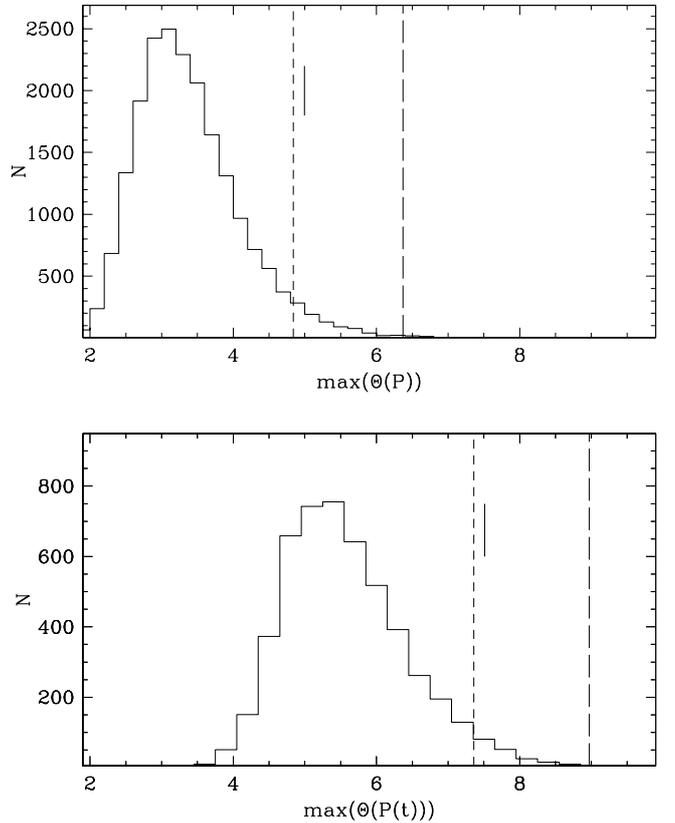


**Fig. 5.** The  $\epsilon - P_{\text{orb}}$  relation for SUUMa systems. The three dashed lines represent the 1, 2,  $3\sigma$  regions around the linear fit. For the orbital period of U Gem the  $3\sigma$  error of the linear fit corresponds to  $4.83 \text{ c/d} \leq f \leq 5.24 \text{ c/d}$ .

longer the original superhump, but the alleged additional superhump at  $f \sim 5.55$ . Using a broad range of periods ( $3 \text{ c/d} \leq f \leq 7 \text{ c/d}$ ), we obtain that this signal is even less significant than the alleged normal superhump, i.e.,  $p = 0.38 \pm 0.02$ . As this weak signal is the first additional superhump ever mentioned, no additional information like an established relation between superhump excess and orbital period exists. The only restriction to the range of trial periods we can apply is to use only those periods that in principal could be interpreted as superhumps, i.e.,  $-0.076 \leq \epsilon \leq 0.241$ . This corresponds to  $4.53 \text{ c/d} \leq f \leq 6.11 \text{ c/d}$  and we obtain a reduced p-value of  $p = 0.20 \pm 0.02$  which, however, is still far from representing a statistically significant value. The corresponding distribution of  $\max(\Theta(P))$  is shown in Fig. 7.

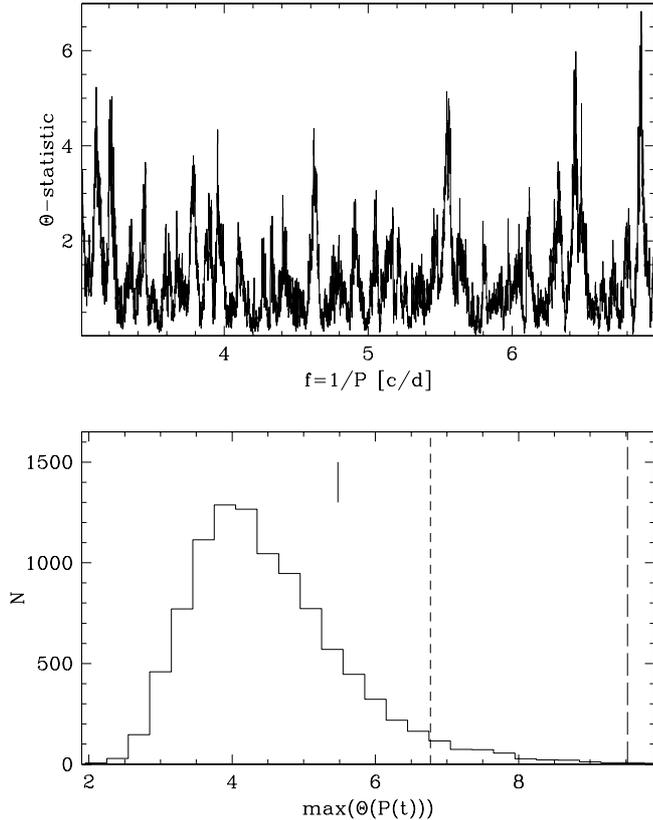
## 6. Summary and conclusion

We analysed the significance of the alleged superhump signal in the famous long 1985 outburst of U Gem using analysis



**Fig. 6.** Distribution of the maximum of the statistic  $\Theta(P)$  for 5 bins and assuming that only periods in the  $3\sigma$  interval of the  $\epsilon - P_{\text{orb}}$  relation can be interpreted as superhumps. In the top panel we used only constant periods, while we took into account coherent time evolution for the superhump periods in the bottom panel. The short solid vertical lines indicate the positions of the claimed superhump signal. The dashed lines represent the 95% and 99.7% ( $3\sigma$ ) significance levels. In both cases the statistical significance of the superhump signal is reaching  $2\sigma$ , i.e.,  $p = 0.0381 \pm 0.001$  (top panel) and  $p = 0.0384 \pm 0.003$  (bottom panel).

of variance (AoV) and Monte-Carlo techniques. As randomisation methods may fail if the observations are correlated,



**Fig. 7.** AoV periodogram for the last 40 measured magnitudes covering JD = 2 446 366–22 122 446 382 (*top panel*). The peak at  $f \sim 5.5$  c/d has been interpreted as an “additional superhump” by Smak & Waagen (2004). However, analysing just the last 40 data points using our Monte-Carlo code shows that this signal is not statistically significant. In the distribution of  $\max(\Theta(P))$  (*bottom panel*), the signal is far from reaching  $2\sigma$ , instead  $p = 0.20 \pm 0.02$ .

we derived semi-analytical estimates in parallel. Following Press et al. (1992) and Paltani (2004) we find perfect agreement with our Monte-Carlo results. The results of our analysis are:

- Using only the information provided by the 160 AAVSO data points of the 1985 outburst of U Gem we do not find a statistically significant periodicity. The probability of obtaining a signal similar to the one detected by Smak & Waagen (2004) by chance is  $p \gtrsim 0.2$ .
- If we restrict our numerical analysis to trial periods consistent with the observed  $\epsilon - P_{\text{orb}}$  relation for SU UMa systems we find that the alleged superhump signal is statistically significant. The p-value decreases to  $p \lesssim 0.04$  and the significance is above  $2\sigma$ . In other words, the probability of detecting a periodic signal as strong as the observed one, which is also consistent with the  $\epsilon - P_{\text{orb}}$ , by chance is less than 0.04.
- The alleged additional superhump (Smak & Waagen 2004) is statistically not significant, i.e.,  $p = 0.20 \pm 0.02$ .

In addition to our detailed statistical analysis one should keep in mind that the superhump signal is a typical one not

only because of the agreement of the  $\epsilon - P_{\text{orb}}$  relation but also because of its (constant) amplitude, its appearance 2–3 days after maximum, and its disappearance slightly before the end of the outburst. In this sense, the determined statistical significance of  $2\sigma$  can be considered as a lower limit. On the other hand, one should also be aware of the fact that extending the observed linear correlation between superhump excess and orbital period towards longer orbital periods represents an *assumption* that is not necessarily true. However, balancing the pros and cons, we recommend assuming, as the new working hypothesis, that the mechanisms causing superhumps and superoutbursts in SU UMa systems probably also triggered the 1985 superoutburst and – keeping in mind the weak statistical significance – superhumps in U Gem. Concerning the triggering of superoutbursts, the enhanced mass transfer model is a very promising alternative to the TTI and its predictions are in agreement with the observations of U Gem (Lasota 2001; Smak 2005). In this scenario, the remaining big problem is a missing explanation of superhumps.

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