A study of the Keplerian accretion disk and precessing outflow in the massive protostar IRAS 20126+4104

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Abstract. We report on interferometric observations at 3.2 and 1.3 mm of the massive young stellar object IRAS 20126+4104 obtained in the C⁴⁴S and CH₃OH lines and in the continuum emission. The C⁴⁴S data confirm the existence of a Keplerian disk, as already suggested by various authors. However, the mass of the central object is ∼ 7 M☉, significantly less than previous estimates. We believe that such a discrepancy is due to the fact that the rotation curve is affected not only by the star but also by the mass in the innermost regions of the disk itself: this leads to an overestimate of the stellar mass when low-density tracers are used to study the velocity field over regions larger than a few seconds of arc (i.e. a few 0.01 pc). On the basis of the line profiles we speculate that accretion onto the star might be still occurring through the disk. This seems consistent with current models of high-mass star formation which predict an accretion luminosity equal to that of IRAS 20126+4104 for a 7 M☉ protostar. The CH₃OH lines trace both the disk and the bipolar outflow previously detected in other molecules such as HCO⁺, SiO, and H₂. New H₂ images obtained at 2.2 µm confirm that the outflow axis is undergoing precession. We elaborate a simple model that suitably fits the data thus allowing derivation of a few basic parameters of the precession.

Key words. stars: formation – ISM: jets and outflows – ISM: individual objects: IRAS 20126+4104

1. Introduction

Both theory and observations suggest a model for star formation which proceeds through inside-out collapse of a molecular core and accretion onto a protostellar nucleus. The fate of such a protostar however depends on the amount of material available for accretion. If this reservoir is quickly exhausted, then the protostar slowly contracts until nuclear reactions switch on and the star reaches the zero-age main sequence (ZAMS). Otherwise accretion goes on and the protostar grows till the critical mass is reached beyond which its contraction becomes faster than accretion: this should happen at about 8 M☉ (Palla & Stahler 1993). At this point, the protostar collapses and ignites hydrogen burning reaching the ZAMS: the luminosity thus generated prevents further accretion due to the powerful radiation pressure exerted on the surrounding material. This scenario has the consequence that massive (i.e. early-type) stars should not form through accretion. Such a conclusion is the consequence of the simplistic treatment of the star formation process outlined above. Accretion rates sufficiently large (≥10⁻³ M☉ yr⁻¹) to overcome radiation pressure cannot be excluded. Moreover, magnetic fields and angular momentum conservation likely cause flattening of the collapsing core and hence formation of circumstellar accretion disks which are much less affected by radiation pressure. As a matter of fact, such disks have been found in T Tauri stars (see e.g. Simon et al. 2001), but it is of crucial importance to prove their existence also in association with high-mass stars.

With this in mind, we conducted a detailed study of a luminous (∼10⁴ L☉) young stellar object (YSO) located at a relatively small distance (1.7 kpc): IRAS 20126+4104. This object was first identified in the IRAS point source catalogue on the basis of its far infrared colours typical of ultracompact HII regions (according to the classification by Wood & Churchwell 1989) and of its association with water maser emission (Comoretto et al. 1990). Tofani et al. (1995) performed VLA observations at 1.3 and 3.6 cm towards this source: no continuum emission was detected but it could be established that the H₂O maser spots lie along a SE–NW direction, with velocity increasing from NW to SE. Subsequent single-dish observations by Cesaroni et al. (1999b) demonstrated that the IRAS source and associated H₂O masers are located inside a dense, parsec-scale molecular clump, as already suggested by
Table 1. Frequency set-up for the PdBI observations. SSB tuning was used at 3.2 mm and DSB at 1.3 mm.

<table>
<thead>
<tr>
<th>Line</th>
<th>Centre frequency (MHz)</th>
<th>Bandwidth (MHz)</th>
<th>Channel spacing (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C$^{34}$S(2–1) and CH$_3$OH(2–1) $\nu_1 = 1$</td>
<td>96.356</td>
<td>40</td>
<td>0.078125</td>
</tr>
<tr>
<td>C$^{34}$S(2–1) and CH$_3$OH(2–1) $\nu_1 = 1$</td>
<td>96.356</td>
<td>320</td>
<td>2.5</td>
</tr>
<tr>
<td>CH$_3$OH(2–1) $\nu = 0$</td>
<td>96.749</td>
<td>40</td>
<td>0.078125</td>
</tr>
<tr>
<td>CH$_3$OH(2–1) $\nu = 0$</td>
<td>96.551</td>
<td>160</td>
<td>1.25</td>
</tr>
<tr>
<td>CH$_3$OH(2–1) $\nu_1 = 1$</td>
<td>96.691</td>
<td>160</td>
<td>1.25</td>
</tr>
<tr>
<td>C$^{34}$S(5–4)</td>
<td>241.017</td>
<td>40</td>
<td>0.078125</td>
</tr>
<tr>
<td>C$^{34}$S(5–4) and CH$_3$OH(5–4) $\nu_1 = 1$</td>
<td>241.144</td>
<td>320</td>
<td>2.5</td>
</tr>
<tr>
<td>C$^{34}$S(5–4) and CH$_3$OH(5–4) $\nu_1 = 1$</td>
<td>240.884</td>
<td>320</td>
<td>2.5</td>
</tr>
</tbody>
</table>

the results of Estalella et al. (1993), and confirmed the existence of a bipolar outflow oriented N–S, previously detected by Wilking et al. (1990). Follow-up interferometric observations (Cesaroni et al. 1997, hereafter C97; Cesaroni et al. 1999a, hereafter C99) revealed an unexpected complexity, making of IRAS 20126+4104 the most promising candidate of a massive protostar with a disk-jet system. Surprisingly, the outflow structure revealed in the high-angular resolution maps is dramatically different from that seen on a larger scale: not only the orientation is NW–SE instead of N–S, but the velocity of the lobes reverses so that, e.g., the blue-shifted gas is located to the SE at low velocities, whereas it appears to the NW at high velocities. Such a velocity reversal has remained unexplained to date: in Sect. 4.2.2 we propose a solution to this problem. On the other hand, the change of orientation of the flow axis from the large to the small scale has been interpreted by Shepherd et al. (2000; hereafter SYBT) in terms of precession of the outflow axis from the SE at low velocities, whereas it appears to the NW at high velocities. Such a change of orientation is NW–SE instead of N–S, but the velocity of the lobes reverses so that, e.g., the blue-shifted gas is located to the SE at low velocities, whereas it appears to the NW at high velocities.

In view of the results obtained so far, IRAS 20126+4104 was observed in July 2001 using the 3.5-m Telescopio Nazionale Galileo (TNG) equipped with the near infrared camera spectrograph (NICS, Baffa et al. 2001). A region of approximately 4 × 4 arcmin centered on the IRAS source was imaged in the H$_2$ ($\lambda = 2.12 \mu$m) narrow band filter and in a nearby narrow-band continuum filter, $K_{cont}$, a detailed characterization of all these filters can be found in Ghinassi et al. (2002). A series of dithered exposure were obtained in each filter and used to remove the sky emission. The final integration times per sky position was 30 min in each of the H$_2$ and $K_{cont}$ filters. Data reduction and analysis were performed using the IRAF software package, following standard flat-fielding procedures.
and sky subtraction, the individual images were registered and the final mosaic was produced. No flux calibration was attempted as the night was not photometric. The continuum emission was removed from the H$_2$ image using the $K$$_c$ image, the procedure works very well in most cases except for the brightest or steep spectrum stellar sources which left a positive or negative residual. Accurate ($\leq 0.5$") astrometry was derived for both mosaics using stellar positions from the 2MASS second incremental data release. The angular resolution of the observation was limited to $\sim 0.9$" by the seeing conditions.

### 3. Results

Both C$^{34}$S lines, all ground state transitions of CH$_3$OH(2–1), and most of the vibrationally excited lines of CH$_3$OH(2–1) and (5–4) covered by our frequency setup have been detected. As for CH$_3$CN (see C97 and C99), also the C$^{34}$S and CH$_3$OH lines arise from a compact molecular core of $\sim 1000$ AU, also traced by the continuum emission at 3.2 and 1.3 mm. This can be seen in Fig. 1, where maps of the integrated emission in these lines are superposed on the corresponding continuum emission. It is worth noting that only $\frac{1}{3}$ of the C$^{34}$S emission measured with the 30-m telescope by Cesaroni et al. (1999b) is imaged in the PdBI observations, which indicates that most of the emission originates from a region much larger than the $\leq 1$” core.

In Table 3 we list the parameters of the continuum emission. The angular diameters ($\theta$) have been obtained with Gaussian deconvolution from the full width at half power (FWHP) of the maps; $D$ and $S_v$ indicate respectively the linear diameter and the integrated flux over the whole emitting region.

In Fig. 2 we show the spectra of the C$^{34}$S(2–1) and (5–4) lines obtained integrating the emission over the core, while the spectra of the ground state and vibrationally excited CH$_3$OH transitions are presented in Figs. 3 and 4. The most interesting feature is the fact that all lines observed with sufficiently high spectral resolution present asymmetric profiles, with a sharper cut-off on the blue-shifted side. One possibility is that this is due to red-shifted self-absorption, as suggested by the dip at $\sim -1$ km s$^{-1}$ in the C$^{34}$S(2–1) profile. We shall come back to this point in Sect. 4.3.

Since one of the goals of the present study is to use C$^{34}$S to investigate the disk previously detected in CH$_3$CN and NH$_3$, it is worth comparing the distribution of the red- and blue-shifted C$^{34}$S emission with the bulk of the molecular gas traced by the millimeter continuum. This is done in Fig. 5 for both C$^{34}$S lines: there is little doubt that the gas is distributed in a NE-SW elongated structure consistent with the orientation (PA = 53$^\circ \pm 8^\circ$) of the plane of the disk detected in CH$_3$CN (see C99). A similar result holds for all transitions detected in this study, thus confirming that such a disk has a rich chemical composition.

On the other hand, the ground state CH$_3$OH lines do not arise only from the disk, but also from the material ejected along the disk axis. This is shown in Fig. 6, where a map of the emission averaged under all of the CH$_3$OH(2–1) ground state lines is compared to the jet revealed in the SiO(2–1) (from C99) and H$_2$ ($v = 0$–1 S(1) transitions. It is interesting to note that Kurtz et al. (2004) have detected several CH$_3$OH maser spots at 44 GHz lying along the border of the thermal CH$_3$OH emission in the jet, as shown in the upper panel of Fig. 6: this is consistent with the idea that class I methanol masers form in shocks. The CH$_3$OH emitting regions seem complementary to those traced by the SiO and H$_2$ molecules, although this effect is much more evident in the SE lobe than in the NW one. Such a complementarity resembles the distribution of the NH$_3$(3, 3) and (4, 4) lines observed by Zhang et al. (1999; see their Fig. 1): as suggested by these authors, this result might be an effect of dissociation of the CH$_3$OH and NH$_3$ molecules in a high-velocity shock, which instead favours the formation of SiO. Such a scenario is also consistent with the lower velocities measured in the CH$_3$OH lines (as discussed in Sect. 4.2) with respect to those estimated by C99 for the SiO transition.

### Table 2. Instrumental parameters for the IRAM PdBI observations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre position</td>
<td>$\alpha$(J2000) = 20$^h$14$^m$26$^s$36</td>
</tr>
<tr>
<td></td>
<td>$\delta$(J2000) = 41$^\circ$13$'$32$''$52</td>
</tr>
<tr>
<td>Number of antennas</td>
<td>6</td>
</tr>
<tr>
<td>Baseline range</td>
<td>48--331 m</td>
</tr>
<tr>
<td>Band centre</td>
<td>96.526 and 241.054 GHz</td>
</tr>
<tr>
<td>Total correlator bandwidth</td>
<td>580 MHz</td>
</tr>
<tr>
<td>Number of sections in the correlator</td>
<td>8</td>
</tr>
<tr>
<td>Primary HPBW</td>
<td>50&quot; at 3.2 mm and 20&quot; at 1.3 mm</td>
</tr>
<tr>
<td>Synthesised HPBW</td>
<td>$\sim 2\frac{1}{4}$ at 3.2 mm and $\sim 0\frac{1}{85}$ at 1.3 mm</td>
</tr>
<tr>
<td>Primary flux density calib.</td>
<td>3C 273: 9.7--11.8 Jy at 3.2 mm; 3.7--7.8 Jy 1.3 mm</td>
</tr>
<tr>
<td></td>
<td>3C 345: 4.7 Jy at 3.2 mm; 4.0 Jy at 1.3 mm</td>
</tr>
<tr>
<td>Phase and amplitude calib.</td>
<td>2013+370; 2.7--3.1 Jy at 3.2 mm; 1.5--3.0 Jy at 1.3 mm</td>
</tr>
</tbody>
</table>

### Table 3. Parameters of the continuum emission: $\theta$ and $D$ are the deconvolved angular and linear diameters and $S_v$ is the integrated flux over the whole emitting region.

<table>
<thead>
<tr>
<th>$\nu$ (GHz)</th>
<th>FWHP (arcsec)</th>
<th>$\theta$ (arcsec)</th>
<th>$D$ (AU)</th>
<th>$S_v$ (Jy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>95.01</td>
<td>2.54</td>
<td>0.92</td>
<td>1570</td>
<td>0.022</td>
</tr>
<tr>
<td>239.55</td>
<td>1.03</td>
<td>0.55</td>
<td>935</td>
<td>0.181</td>
</tr>
</tbody>
</table>
Fig. 1. Contour maps of the $^{34}$S and CH$_3$OH emission integrated under the line for the transitions detected towards IRAS 20126+4104 in the PdBI observations. Also shown are the maps of the millimeter continuum at the corresponding frequency (grey scale). The three CH$_3$OH maps have been obtained averaging the emission under the ground state CH$_3$OH(2–1) $\nu_1 = 1 + 1\ E$ and 0 E lines, and the CH$_3$OH(5–4) $\nu_1 = 1 + 4\ A$, -3 E, -4E, -2E, 2 A, 3 A, 1 E, 0 E, and 2 E lines. The cross marks the position of the millimeter continuum peak. Contour levels range from 0.02 to 0.11 by 0.015 Jy/beam for $^{34}$S(2–1), from 0.02 to 0.3 by 0.14 Jy/beam for CH$_3$OH(2–1), from 0.01 to 0.11 by 0.02 Jy/beam for CH$_3$OH(2–1) $\nu_1 = 1$, from 0.05 to 0.37 by 0.08 Jy/beam for $^{34}$S(5–4), and from 0.08 to 0.73 by 0.13 Jy/beam for CH$_3$OH(5–4) $\nu_1 = 1$. The synthesised beam full widths at half power (position angles) are the following (from top to bottom): $\times 7' \times 2' 1$ (44$^\circ$), $\times 7' \times 2' 6$ (79$^\circ$), $3'1 \times 2' 0$ (35$^\circ$), $0'87 \times 0'75$ (136$^\circ$), $0'87 \times 0'75$ (137$^\circ$).

Fig. 2. Spectra obtained by integrating the $^{34}$S line emission over the whole region where the corresponding transition is detected above a 3$\sigma$ level. The dashed and dotted lines indicate respectively the systemic velocity of the source and the zero flux level.

Fig. 3. Same as Fig. 2 for the ground state lines of CH$_3$OH. The dotted lines mark the frequencies of each CH$_3$OH transition.

4. Discussion

In the following, we present an analysis of the disk-outflow system in IRAS 20126+4104 with the aim of giving an answer to the major questions left open by the previous studies of this object. On the basis of such studies one can identify three
components in this object: the disk (≤3.5′′ or ≤0.03 pc), the outflow/jet, and the envelope (≤10 or 0.08 pc). Since the main goal of our study is to investigate the former two (which will be discussed in Sects. 4.1 and 4.2), it is important to establish which is the contribution of each component to the tracers used in our analysis. For this purpose we adopt a criterion based on the kinematics and morphology of the emitting region. In fact, while all three components may contribute to the line emission close to the systemic velocity, in the line wings the contribution of the envelope is very likely negligible. Moreover, the direction of the velocity gradient in the disk is almost perpendicular to that in the outflow/jet, which makes it easy to discriminate between the two.

In practice, for a given line it will be sufficient to draw two position–velocity plots, one along the plane of the disk and the other along the axis of the flow, and check for the presence of velocity gradients in these two directions. For example, let us consider the case of CH$_3$OH. In Fig. 6 one can clearly see that the emission arises from both the outflow lobes and the central core. In Sects. 4.1 and 4.2, we shall demonstrate that a velocity gradient is found in both the outflow and the disk direction. We may hence conclude that CH$_3$OH is tracing both the disk and the outflow component. The same method can be applied to all lines, leading to the conclusion that the disk is seen in C$^{34}$S, CH$_3$CN, CH$_3$OH, and HCO$^+$, whereas the outflow/jet is detected in HCO$^+$, CH$_3$OH, and SiO.

What about the envelope? The distinction between this and the disk is not as sharp as that between the outflow and the disk because the disk may be reasonably seen as the innermost part of the envelope that has undergone flattening and rotation due to conservation of angular momentum. Nevertheless, for the purpose of simplifying the description of the environment of IRAS 20126+4104, it is convenient to discriminate between the more quiescent material surrounding the disk and the rotating disk itself. In this simplified scenario, the envelope is important only at low velocities, where also part of the disk and outflow may significantly contribute. Morphologically, it must be located around the YSO position but this is also the case for the disk and – to some extent – for the outflow. It is hence very difficult to establish which fraction of the line or continuum emission is coming from the envelope, unless no
emission is detected towards the YSO, as in the case of SiO (see top panel of Fig. 6) which may be taken as a “pure” outflow/jet tracer. Such a problem is even more important for the continuum emission for which the velocity information is obviously missing. This “confusion” between components in the central region is especially relevant for the disk, as the outflow extends over a much larger region. For a given tracer, it is hence very difficult to find out what fraction of the emission is arising from the disk and which from the envelope. We conclude that it is safer to consider any mass estimate of the disk as an upper limit.

4.1. The disk

As previously mentioned, evidence for a Keplerian disk in IRAS 20126+4104 has been presented by various authors. In particular, the CH$_3$OH data of C99 made it possible to obtain the direction of the plane of the disk by fitting the peak distribution in their Fig. 5b: the resulting position angle is $53^\circ \pm 7^\circ$, implying a position angle of $-37^\circ \pm 7^\circ$ for the disk axis. This is significantly different from the direction of the outflow/jet axis (PA = $-60^\circ$), obtained by the same authors from the SiO and H$_2$ maps of the jet (see also Fig. 6).

The observations obtained so far were hindered by poor spectral or angular resolution, which prevented direct imaging of the disk geometry and velocity field, and hence an accurate estimate of the stellar mass. On the contrary, the new data obtained by us in the C$^{18}$S lines make it possible to resolve the disk both spatially and in velocity, as illustrated by the position–velocity plots in Fig. 7, where the offset is computed along the plane of the disk corresponding to the NE–SW line in Fig. 5. In the following (see Sect. 4.1.3) we present a model fit to these plots, but even a qualitative inspection of Fig. 7 is sufficient to show that our findings are in qualitative agreement with Keplerian rotation. For this purpose, we have drawn the borders of the region inside which emission is expected for an edge-on Keplerian disk of radius 4.5'' (0.037 pc) rotating about a 7 $M_\odot$ star: this pattern agrees reasonably well with the data, taking into account that it is obtained for zero line width and infinite angular resolution.

In the light of these results, we have then re-examined the data in C97 and C99, which are compared to the new C$^{18}$S data in Figs. 8 and 9. In all cases the Keplerian pattern is evident, with the exception of the CH$_3$CN $K = 8$ and CH$_3$OH lines, detected only over a region close to the position of the star. HCO$^+$ and its isotopomer trace larger radii – and hence lower velocities – in the disk, with respect to the other molecules. This may be due to variations in the molecular abundances of the different species. Another possibility is that one is observing an excitation effect due to density and/or temperature gradients in the disk. A hint in this sense is given by the high energy (513 K) CH$_3$CN(12–11) $K = 8$ line (see Fig. 9) which, unlike the low energy (87–122 K) $K = 2$ and 3 lines, presents two distinct peaks of emission: one close at the systemic velocity and close to the centre, the other slightly offset from the center and at $\sim$5 km s$^{-1}$ with respect to the systemic velocity. The fact that a higher excitation transition arises from a small region close to the star supports the existence of a temperature increase towards the centre of the disk, as already suggested by C99. In the following section we investigate this possibility in some better detail and derive an estimate of the density and its distribution in the disk. On the other hand, the secondary peak in the CH$_3$CN(12–11) $K = 8$ line could be due to a stellar companion of the (proto)star located at the centre of the disk. Although no other evidence of the existence of such a companion has been found, this cannot be ruled out a priori given that binary (or multiple) systems are more common than single stars. The presence of such a system could also explain the precession of the jet in IRAS 20126+4104 revealed by SYBT (see also Sect. 4.2.1). It is also worth noting that the secondary peak in the bottom panel of Fig. 9 lies on the Keplerian curve plotted in Fig. 7, which demonstrates that the velocity and position of such a peak are consistent with those of a lower mass companion orbiting around a 7 $M_\odot$ star.
Position–velocity plots along the plane of the disk (see Fig. 5) for the CH$^+$ of $5 \times 10^{-3}$ cm$^{-3}$ (equal to the ratio between the line width $\Delta v$ and the systemic velocity). In the disk plane the density is large enough (see below) that CH$^+$ molecules may be assumed in LTE, hence an estimate of the mean temperature over the disk measured at a given velocity $v$ is

$$
\langle T \rangle_v = \frac{3T_0}{2q + 3} \left( \min \left\{ \frac{1}{|v|}, \frac{1}{|v|} \right\} \right)^{\frac{q}{3}}
$$

with $v_0$ rotation velocity at radius $R_0$. This expression may be fitted to the curve in Fig. 11b: the best fit, represented by the dashed line in the same figure, is obtained for $T_0 = 67$ K, $v_0 = 2.3$ km s$^{-1}$, and $q = -0.57$. For a geometrically thin accretion disk, no matter whether passive (i.e. heated by the central star) or active (i.e. heated internally by viscosity), theory has established that the temperature profile is a power law with $q = -0.75$ (Pringle 1981): the latter value is consistent with that found by us, within the uncertainties of our method.

We also address the problem of the density distribution in the disk. The ratio between the C$^{34}$S(5–4) and (2–1) lines can be used to calculate the H$_2$ density, as explained, e.g., in Cesaroni et al. (1991). Figure 12b shows a position–velocity plot of such a ratio along the plane of the disk, obtained after smoothing the C$^{34}$S(5–4) data to the same angular resolution as the (2–1) line. In order to convert this into a density estimate, we used the LVG code of Cesaroni et al. (1991), adopting a C$^{34}$S abundance relative to H$_2$ of $5 \times 10^{-10}$, a velocity gradient of 13 km s$^{-1}$ pc$^{-1}$ (equal to the ratio between the line width and the disk diameter), and a kinetic temperature of 170 K (obtained from CH$_3$OH). In general for any given temperature and line ratio, two distinct values of the density are possible. This is illustrated in Fig. 13, where such a ratio is plotted as a function.
Fig. 8. Same as Fig. 7 for the C$^{34}$S(2–1), CH$_3$OH(2–1)$_E$, CH$_3$OH(2–1)$_V$, CH$_3$OH(1–0)$_V$, HCO$^+$(1–0) (from C97), and H$^{13}$CO$^+$(1–0) (from C99) lines. Contour levels range from 0.02 to 0.14 by 0.02 Jy/beam for C$^{34}$S, from 0.035 to 0.28 by 0.035 Jy/beam for CH$_3$OH, from 0.02 to 0.06 by 0.01 Jy/beam for CH$_3$OH $V = 1$, from 0.08 to 3.08 by 0.5 Jy/beam for HCO$^+$, and from 0.012 to 0.087 by 0.015 Jy/beam for H$^{13}$CO$^+$. 

of temperature and density. For instance, one can see that, for $T_K = 170$ K, a line ratio of 1 – the mean value in Fig. 12b – is attained for either $n(H_2) \sim 2.5 \times 10^5$ cm$^{-3}$ or $\sim 2 \times 10^6$ cm$^{-3}$. Inspection of Fig. 12b suggests which of the two values must be chosen. As already pointed out for the temperature plot in Fig. 12a, going from low to high velocities corresponds to moving from the outer to the inner regions in the disk: correspondingly, the line ratio increases from $\sim 1$, to a maximum of $\sim 1.8$, and eventually decreases to $\sim 1$ at high velocities. Very likely this means that the density increases steadily outside in, starting from $\sim 2.5 \times 10^5$ cm$^{-3}$ at the outer radius and reaching values $\sim 2 \times 10^6$ cm$^{-3}$ close to the centre.

4.1.2. Mass

The mass of the disk can be derived from its millimeter continuum emission. Such an estimate depends on the temperature of the dust and the composition of the grains. In order to evaluate the best possible value of the mass, we have considered all interferometric measurements available in the literature at millimeter wavelengths, where the emission is optically thin: these are plotted in Fig. 14 as a function of frequency. Note that the scatter of the values at 1 mm is resolved if only the uv-points in common between the different data sets are used for the image reconstruction. The spectrum in this regime can be described by a power law with index $\sim 2.7$, which implies a dust absorption coefficient $\kappa \propto \nu^0.7$. Under the assumption of thermal equilibrium between gas and dust at 170 K (see Fig. 10), one obtains a mass ranging from 0.65 to 3.6 $M_\odot$ depending whether one adopts the dust opacities of Hildebrand (1983), Ossenkopf & Henning (1994), or Kramer et al. (2003).

An independent estimate of the mass is obtained from the CH$_3$OH column density integrated over the source itself, using the results of Sect. 4.1.1. In hot-core regions the CH$_3$OH abundance relative to H$_2$ is found to vary between $10^{-7}$ and $10^{-6}$ (see Van Dishoeck 1993). Consequently, one estimates a disk mass between 0.8 and 8 $M_\odot$.

Given the large uncertainty on the CH$_3$OH abundance, we consider more reliable the value obtained from the continuum emission, thus concluding that the mass of the disk within $\sim 5000$ AU from the star very likely lies in the range 1–4 $M_\odot$.

4.1.3. Velocity field and stellar mass

As we mentioned earlier, the position–velocity plots of Fig. 7 represent solid evidence of Keplerian disk rotation and we now further discuss this critical point. In fact, the spectral and angular resolution in our observations were good enough to allow an accurate study of the velocity field inside the disk. This analysis is of fundamental importance to assess the correct value of the stellar mass and hence shed light on its nature and evolutionary phase. The basic procedure to determine the mass of the
central star(s) consisted of using all of the data in the position–velocity plot of Fig. 7 at once, by least-squares fitting the maps for several kinematical and physical parameters of the modeled keplerian disk.

In our model we consider an edge-on cylindrical disk undergoing Keplerian rotation. The disk has an outer, \( R_0 \), and inner, \( R_i \), radius, and the star is located at the center of the inner (cylindrical) cavity. The temperature and volume density are assumed to depend on the distance from the centre, \( R \), respectively as \( T \propto R^q \) and \( n \propto R^p \). The radiative transfer equation was solved along a suitable number of lines of sight in the disk plane, taking into account the appropriate velocity coherence paths. Finally, the resulting line brightness temperature was convolved with the synthesized beam and with the velocity resolution of our PdBI observations, allowing the generation of a model position–velocity plot to be compared with the data.
Fig. 12. a) Position–velocity plot along the plane of the disk for the rotation temperature (grey scale) obtained from the ratio between the same CH$_3$OH(2–1) lines as in Fig. 11. For the sake of comparison, also a contour map of the C$^{34}$S(2–1) line emission is shown; contour levels range from 0.03 to 0.15 by 0.03 Jy/beam. b) Same as above, for the ratio (grey scale) between the C$^{34}$S(5–4) and C$^{34}$S(2–1) lines.

The least-squares fitting of the map then followed three steps. First, the model data, obtained with the procedure outlined above, were resampled on the same grid points as the observed data. Then, for each selected mass of the central star(s) the least-squares procedure found the best-fit parameters. The rotation velocity at the outer radius is not an independent parameter but it is obtained from the star mass and $R_0$. The value of $R_0$ was assumed to be the same for both C$^{34}$S lines. Finally, several trial masses were used and we found that the best results were achieved for a mass of 7 $M_\odot$. Besides the mass of the central star, the best-fit parameters obtained with this procedure are the line width ($\Delta V$), the excitation temperature at the outer radius of the disk ($T_{ex}$), the peak column density of the C$^{34}$S molecule ($N_{C^{34}S}$), and the power-law index for the density gradient ($p$). The power-law index for the temperature profile was instead fixed at $q = -0.75$ (see Sect. 4.1.1). The best fits have been obtained for the parameters listed in Table 4 and are compared to the data in Fig. 15.

Although the pattern is reproduced reasonably well, both in the overall shape and intensity, a few caveats are in order. First of all, C$^{34}$S is unlikely to be in LTE, so that the values of $T_{ex}$ and $N_{mol}$ used in the computation are to be regarded as a way to parameterise the excitation conditions and not as true physical parameters. This explains why the fit to the C$^{34}$S(2–1) line is obtained for slightly different parameters than that to the C$^{34}$S(5–4) line. Moreover, the density gradient describes the variation of the C$^{34}$S density, which does not necessarily resembles the H$_2$ density change if the C$^{34}$S abundance depends on the radius. Due to these limitations, most of the physical quantities obtained from the fits are to be regarded with caution and are certainly affected by large errors. The sole exception is represented by the mass of the star, which is quite well constrained by the shape of the position–velocity plots: in particular, a small variation in the velocity of the “spike” corresponding to the largest radius where emission is detected will cause a significant change in the stellar mass. One may thus estimate the uncertainty on the value of the mass, assuming an error of 0.15 km s$^{-1}$ (half the spectral channel used in the position–velocity plots) and 0'4 (half the beam size at 1.3 mm): for a

### Table 4. Parameters for the model fits in Fig. 15.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>HCO$^+$(1–0)</th>
<th>C$^{34}$S(2–1)</th>
<th>C$^{34}$S(5–4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_*$ ($M_\odot$)</td>
<td>12</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>$R_0$ (arcsec)</td>
<td>7.1</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>$\Delta V$ (km s$^{-1}$)</td>
<td>0.40</td>
<td>0.40</td>
<td>0.45</td>
</tr>
<tr>
<td>$T_{ex}$ (K)</td>
<td>50</td>
<td>40</td>
<td>30</td>
</tr>
<tr>
<td>$p$</td>
<td>-2.1</td>
<td>-3.0</td>
<td>-2.8</td>
</tr>
<tr>
<td>$N_{mol}$ (10$^{12}$ cm$^{-2}$)</td>
<td>0.22</td>
<td>17</td>
<td>8.4</td>
</tr>
</tbody>
</table>
Keplerian orbit passing through a point at 2′′.2 ± 0′′.4 with velocity 1.3 ± 0.15 km s⁻¹, the resulting central mass is 7 ± 2 M⊙.

This result significantly differs from the estimate of ∼20 M⊙ obtained by C99 and Zhang et al. (1998). The method used by the C99 to calculate the mass of the star is based on rough estimates of the disk diameter and corresponding velocity dispersion, severely affected by the poor spectral resolution available (∼10 times worse than in the present study); one may get an idea of this by comparing the top with the middle panel in Fig. 9. On the other hand, the observations by Zhang et al. (1998) did not suffer by this problem: both angular and spectral resolution are sufficient to resolve the Keplerian pattern (see their Fig. 3) and their model fit seems to reproduce the data reasonably well with a point mass of 20 M⊙. Interestingly, a similar result is obtained from our C34S data (see Fig. 8). Applying our numerical model to such plots (see Fig. 15), we find a reasonable fit for a stellar mass of about 12 M⊙, significantly greater than that needed to fit the C34S data.

However, both the ammonia maps of Zhang et al. (1998) and the HCO+ maps of C97 and C99 sample regions at least twice as large as that sampled by our C34S observations: this suggests that part of the mass required by the “Keplerian” fits to the outer disk might be contributed by the inner portion of the disk itself rather than by the star. In fact, Bertin & Lodato (1999) have recently demonstrated that the rotation curve of self-gravitating circumstellar disks follows a Keplerian pattern close to the star but tends to a constant velocity at larger distances, where the disk mass enclosed inside the radius becomes greater than the stellar mass. This might be the case of IRAS 20126+4104. In Sect. 4.1.2 we have estimated a mass of the disk of ≤4 M⊙ inside the C34S region: this is significantly less than the stellar mass. However, inside a region twice as large, the disk mass may increase by a factor ∼4, assuming constant column density: this brings the total mass of the star+disk system to ≤16 M⊙, i.e. closer to the estimate obtained from NH3 and HCO+.

In conclusion, we believe that our estimate of ∼7 M⊙ is a reasonable value for the mass of the central (proto)star.

4.2. The outflow/jet

The existence of a bipolar outflow/jet in IRAS 20126+4104 has been assessed by many authors in a variety of tracers over scales ranging from 100 AU to 0.5 pc. Since the jet and the outflow are manifestations of the same ejection phenomenon, for the sake of simplicity in the following we shall use only the word “outflow” to indicate both the jet detected in H2, SiO, CH3OH, and NH3, and the outflow seen in CO, HCO+, and their isotopomers.
The knowledge about the outflow in IRAS 20126+4104 prior to the present study may be summarised as follows:

- The opening angle of the flow is 21° and its axis forms an angle of 9° with the plane of the sky, with the north-western part lying in the halfspace of the observer with respect to the plane of the sky (see C99).
- The position angle of the outflow axis remains equal to \(\sim -60°\) at a distance from the origin ranging from 0′1 (170 AU) to 10′′ (0.08 pc) (Moscadelli et al. 2000; Hofner et al. 1999; Zhang et al. 1999; C97; C99; Kawamura et al. 1999), but becomes \(\sim 0°\) over a region of \(\sim 1′\), as demonstrated by SYBT, who interpret this as precession of the outflow.
- The expansion velocity of the flow lies in the range 20–200 km s\(^{-1}\), depending on the distance from the origin, which suggests acceleration of the gas (see Moscadelli et al. 2000 and C99).
- A study of the emission at different velocities across the HCO\(^+\) and SiO lines reveals that while the blue and red lobes are clearly separated at high velocities (i.e. in the outer wings of the line) with the blue-shifted emission located to the NW and the red-shifted to the SE, at low velocities the red- and blue-shifted lobes overlap (see C97 and C99).

The situation is outlined in Figs. 6 and 16. The latter presents the position–velocity plots along the outflow axis for the new CH\(_3\)OH (2–1) data (contours) and the HCO\(^+\) (1–0) and SiO (2–1) transitions (grey scales) observed by C99. The overlap between blue and red lobes mentioned above stands out clearly in this figure: while close to the systemic velocity no clear positional separation is seen between blue- and red-shifted emission, at high-velocities the red-shifted gas is present only to the SE and the blue-shifted to the NW. This effect has been convincingly explained by C99 with the fact that the outflow cone crosses the plane of the sky (see their Fig. A1 for a sketch of the model). The gas close to the plane of the sky moves almost perpendicularly to the line of sight and lies on both halfspaces with respect to the plane of the sky: consequently it is seen at low velocities and contributes to both red- and blue-shifted emission in the same lobe. The remaining material lies in only one halfspace and has a larger velocity component along the line of sight: it is hence seen only at either red- or blue-shifted high velocities.

However, this model cannot explain why the intensity of the low-velocity HCO\(^+\) emission is not symmetric with respect to the systemic velocity (see upper panel in Fig. 16). In fact, the low-velocity pattern presents an S-shaped appearance so that the emission to the SE peaks at blue-shifted velocities, whereas that to the NW is clearly red-shifted. Such an asymmetry cannot be explained with the homogeneous conical model of the flow adopted by C99. Although less prominent, the same pattern is visible also in the position–velocity plots of CH\(_3\)OH (see contours in Fig. 16) and NH\(_3\) (see Fig. 3 of Zhang et al. 1999).

An alternative to the position–velocity plot of Fig. 16 are the HCO\(^+\) outflow maps presented in Fig. 17, where – following C97 – the emission in the inner wings of the line (low velocity) is compared to that in the outer wings (high velocity). Note that the maps have been rotated by 37° to align the disk rotation axis to the y-axis of the figure. For the sake of comparison, also the C\(^{34}\)S disk and SiO flow are shown. The S-shaped feature in the HCO\(^+\) position–velocity plot has the consequence that the low-velocity gas is dominated by red-shifted emission in the NW lobe and by blue-shifted emission in the SE lobe, whereas in a homogeneous outflow the blue- and red-shifted gas should contribute almost equally to both lobes at low velocities.

A naïve explanation of this effect is that the S-shaped pattern is due to a second outflow with a different inclination with respect to the line of sight. Although it is impossible to rule out this possibility, we also present an alternative scenario which requires only one outflow and satisfies all the observational evidence collected so far. The key to interpret the S-shaped pattern in the HCO\(^+\) position–velocity plot is the precession of the outflow discovered by SYBT. Therefore, before discussing the S-shaped feature, we elaborate a model to describe such a precession.
4.2.1. A model for the precessing outflow

The best tracers to identify this phenomenon are the H$_2$ and H$_\alpha$ knots detected by C99 and SYBT over a region as large as $\sim 2''$ ($\sim 1$ pc). These show the projection onto the plane of the sky of the trajectory drawn by a bullet shot along the precessing axis of the flow. Therefore, the knots must lie over the surface of the cone described by the outflow axis during its precession. In our model we assume that the knots have radial velocities diverging from the vertex of the cone and that the precession occurs with constant angular velocity about the rotation axis of the disk. Under these hypotheses, the model is described by five parameters: the RA and Dec of the vertex, the inclination angle ($\psi$) between the disk axis and the plane of the sky, the opening angle ($\theta$) of the precession cone, and the ejection velocity ($\dot{R}$), which may be expressed as a function of the distance $R$ from the vertex.

The coordinates of the vertex are given by the peak of the millimeter continuum, which is a good approximation to the position of the embedded YSO powering the outflow.

An estimate of $\theta$ is given by the maximum angle between the disk axis and the outflow axis; since the position angle of the former is $-37^\circ$ and the oldest direction of the outflow axis corresponds to the N–S orientation of the $^{12}$CO flow observed by SYBT, very likely $\theta \approx 37^\circ$.

It is unlikely that the disk axis is significantly inclined with respect to the plane of the sky. In fact, if $\psi = 0$ the outflow axis will lie in the plane of the sky when its angle with the disk axis will be equal to the opening angle of the precession cone, i.e. $37^\circ$. Indeed, C99 have demonstrated that the current direction of the outflow axis lies very close ($\sim 9^\circ$) to the plane of the sky and that the projection of such axis onto the plane of the sky forms an angle of $\sim 23^\circ$ with the disk axis, i.e. close to $\theta = 37^\circ$. We may hence assume $\psi = 0$ in the following.

With this in mind, one can estimate the expansion velocity law of the flow. Since the precession angular velocity ($\dot{\phi}$) is constant, the precession angle ($\phi$) can be taken as a measure of time. Moreover, the 3D geometry of the precession cone is known, so that it is possible to compute $\phi$ and $R$, for each knot using the following expressions:

$$
\tan^2 \phi = \frac{y^2 \tan^2 \theta - x^2}{x^2}
$$

$$
R^2 = \frac{y^2}{\cos^2 \theta}
$$

under the assumption of $\psi = 0$ and with $x, y$ reference system centred on the star and lying in the plane of the sky, with $y$ representing the projection of the disk axis. A plot of $R$ versus $\phi$ for the knots in Table 1 of SYBT is shown in the upper panel of Fig. 18. The two quantities are reasonably correlated according
the flow has precessed by at least 1150°. It is worth noting that the outflow age is 10 times greater than the time required by the farthest knot to reach the current position expanding at a speed of 100 km s⁻¹: this suggests that more knots might be found on a larger scale if images with sufficient field of view and sensitivity will be available.

The origin of the precession has been extensively discussed by SYBT and will not be further investigated here. These authors concluded that the precession could be caused by the interaction between the disk and a stellar companion orbiting about the (proto)star at the disk centre. One may speculate that such a companion might be located at the position of the secondary peak seen in the CH₃CN(12–11) K = 8 line (see Sect. 4.1).

4.2.2. The S-shaped low-velocity feature

An important consequence of the fit obtained in the previous section is that at present the outflow axis is about to cross the plane of the sky, with the NW part of it receding from the observer and the SE part approaching him. Since the precession is slow compared to the ejection, the result is that at any given time the material surrounding the outflow lobes is not homogeneously distributed around them: the region left behind in the precession has been swept away by the flow itself, whereas the molecular gas towards which the flow is precessing is still unaffected.

For example, let us consider the NW lobe of the outflow. The blue-shifted, high-velocity emission (see left panel of Fig. 17) is contributed by the part of this lobe that still lies in the halfspace of the observer. Figure 17 illustrates that the SiO flow is aligned along the current direction of the outflow axis (dotted line), while the bow shocks at the head of it mark the place where the ejected material is impinging against the surrounding molecular cloud. The lobe is precessing around the disk axis (thick vertical line) moving from left to right, thus approaching the intersection of the precession cone with the plane of the sky (full line in the left panel of Fig. 17). It is interesting to note that such a precession is revealed by the wake of emission left behind the outflow lobes.

On the other hand, the part of the NW lobe that has crossed the plane of the sky contributes to the red-shifted, low-velocity emission (right panel of Fig. 17), mainly for two reasons: it is moving almost perpendicularly to the line of sight and it is entraining fresh molecular gas which increases the density of the flow, enriches its composition, and brakes the expansion. Consequently, the red-shifted line intensity is boosted by the density increase and molecular enrichment unlike the blue-shifted gas which, lying closer to the evacuated region, is much less dense and subject to higher expansion velocities.

The previous scenario explains the S-shaped asymmetry in the position velocity plots. Also, it solves the paradox that most molecules seem to trace blue-shifted emission to the NW and red-shifted to the SE, while we know that this should not be the case since the outflow axis points towards the observer in the NW part. The explanation is that molecular abundances are much more prominent in front of the precessing lobes.

![Figure 18](image-url)
4.3. Infall and nature of the YSO

What is the nature of the YSO in IRAS 20126+4104? The question we want to address here is whether one is dealing with a ZAMS star or with a protostar. For this purpose it is helpful to compare the luminosity and the mass of the star. The latter has been evaluated in Sect. 4.1.3 as \( \sim 7 \, M_\odot \), while the former is \( \sim 10^4 \, L_\odot \) (C99). The luminosity of a ZAMS with a similar mass should be \( \sim 10^3 \, L_\odot \), an order of magnitude less than that of IRAS 20126+4104: such a discrepancy may be explained if IRAS 20126+4104 is a protostar deriving its luminosity from accretion. In fact, according to the computation of Behrend & Maeder (2001), \( \sim 10^4 \, L_\odot \) is just the total luminosity of a \( \sim 7 \, M_\odot \) protostar plus disk (see their Table 1). In their model, an accretion rate of \( 10^{-3} \, M_\odot \, yr^{-1} \) is required: is this consistent with our observations?

As noted in Sect. 3, in the C\(^{34}\)S spectra one may see a hint of red-shifted self-absorption, suggestive of infall in the disk. As a matter of fact, the presence of infall in massive star forming regions has been observationally established (see e.g. Keto et al. 1988; Ho et al. 1996; Sollins et al. 2004) and theoretically predicted (Keto 2003) as a crucial phenomenon profoundly affecting the formation of early-type stars. In order to estimate the accretion rate, we use the same approach as in C99:

\[
M_{\text{acc}} = \frac{2 \pi}{\alpha^2} R^3 \left( \frac{G \, M_*}{R} \right)^{1/2} \]

where \( M(R) \) is the mass of the disk inside a given radius \( R \). From Table 3 one sees that the mass of the disk (\( \sim 4 \, M_\odot \)) traced by our millimeter continuum observations is enclosed inside a radius of 760 AU. Substituting these values in the previous equation one obtains \( M_{\text{acc}} \sim 2 \times 10^{-3} \, M_\odot \, yr^{-1} \), which agrees well with the value derived above from the luminosity of the source. Such an accretion rate is bound to exhaust the material in the disk after about \( 4 \, M_\odot / 2 \times 10^{-3} \, M_\odot \, yr^{-1} = 2 \times 10^3 \, yr \), a short time compared to the outflow age quoted by SYBT (6.4 \( \times 10^4 \) yr). Since it is reasonable that the disk and outflow are physically related entities, their lifetimes are likely similar to each other: this implies that the disk must be continuously fed through accretion from the surrounding parsec-scale envelope. It must be noted that the accretion rate estimated above is computed under the assumption that all the infalling material will accrete onto the star, whereas theoretical models predict that a fraction of it will contribute to the outflow. However, this fraction is expected to be relatively small, \( \sim 30\text{–}50\% \) (see Behrend & Maeder 2001 and references therein). As a matter of fact, the outflow rate estimated by SYBT is \( 8.1 \times 10^{-4} \, M_\odot \, yr^{-1} \), i.e. \( 40\% \) of \( M_{\text{acc}} \). We hence conclude that \( 2 \times 10^{-3} \, M_\odot \, yr^{-1} \) is very close to the accretion rate onto the star.

5. Summary and conclusions

We have performed observations with the Plateau de Bure interferometer towards the high-mass young stellar object IRAS 20126+4104. New images in the H\(_2\) line obtained with the Telescopio Nazionale Galileo are also presented. Our findings confirm the existence of a Keplerian disk and bipolar outflow oriented roughly perpendicularly to the disk. The outflow axis turns out to be precessing as described by SYBT. A simple model is elaborated to describe such a precession and thus derive the corresponding angular velocity (0.018 deg yr\(^{-1}\)).

The mass of the central YSO is obtained by fitting a numerical model to the position velocity plots of the C\(^{34}\)S lines. This turns out to be \( \sim 7 \, M_\odot \), namely 2–3 times less than the estimates obtained with the same method from the NH\(_3\) (1, 1) (Zhang et al. 1998) and HCO\(^+\) lines. We believe that such a discrepancy may be explained by the fact that the disk mass enclosed in the region traced by the NH\(_3\) HCO\(^+\) is non negligible with respect to the mass of the YSO.

Line ratios of the C\(^{34}\)S and CH\(_3\)OH lines prove the existence of a temperature and density gradient in the disk, while the line profiles indicate that the material of the disk might be infalling onto the central YSO at a rate of \( \sim 2 \times 10^{-3} \, M_\odot \, yr^{-1} \). According to current theoretical models (Behrend & Maeder 2001), this is just the accretion rate expected for a \( 7 \, M_\odot \) protostar with the luminosity of IRAS 20126+4104 (\( \sim 10^4 \, L_\odot \)). We hence conclude that IRAS 20126+4104 is likely a massive protostar in the accretion phase.

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Appendix A: Mean kinetic temperature in a Keplerian disk

In this appendix, we elaborate a simplified analytical expression for the mean kinetic temperature (\( T \)) in the plane of the sky for the material moving in a Keplerian disk, at a given velocity (\( v \)) along the line of sight. In the following we assume that all quantities are normalised with respect to the outer radius of the disk, \( R_0 \), and the rotation velocity at such radius, \( v_0 \); therefore in our notation \( x \) stands for \( x/R_0 \), \( R \) (the distance from the centre of the disk) for \( R/R_0 \), and \( v \) for \( v/v_0 \). Moreover, since the mean value of \( T \) must be positive, independently of the sign of \( x \) and \( v \), in the following we shall replace these quantities by their absolute values, i.e. \( x \) will stand for \( |x| \) and \( v \) for \( |v| \).

Figure A.1 shows the outer border of the disk (corresponding to \( R = 1 \)) and the curves corresponding to points with velocity \( v \) for two cases: \( v > 1 \) (curve to the left) and \( v < 1 \) (curve to the right). It is convenient to express such curves as

\[ x = v \, R^2. \]  

(1.1)

The mean value of \( T \), measured in the plane of the sky over the whole disk, is obtained integrating the temperature along the
Fig. A.1. Loci of points in a Keplerian disk corresponding to a given velocity $v$ along the line of sight. All quantities are normalised with respect to the outer radius of the disk and the corresponding rotation velocity. The curve to the right is for $|v| < 1$ and the one to the left for $|v| > 1$. The thick portion of each curve indicates the pattern over which the integral in Eq. (A.2) must be performed. The maximum value of $x$ attainable for each curve inside the disk ($x_v$) is indicated by the dotted lines. The line of sight is parallel to the $z$-axis.

Substituting Eqs. (A.3) and (A.4) in Eq. (A.2), one can evaluate the integral and thus obtain Eq. (1):

$$\langle T \rangle_v = \frac{T_0}{x_v^{2q+1}} \left( \min\{1, v^{-3}\}\right)^{\frac{1}{3q}}.$$  \hspace{1cm} (A.5)

References

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(SYBT)