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On the generation of Alfvén waves by solar energetic particles

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Abstract. A simple analytical theory of Alfvén waves amplified by streaming solar energetic particles (SEPs) is studied. It is pointed out that a finite time-integrated net flux of energetic protons has to pass each point in space before we can expect Alfvén waves to be significantly modified by the streaming instability. The time-integrated net proton flux needed for the time-integrated wave growth rate (or wave growth, for short) to exceed unity is evaluated. Assuming that protons stream much faster than the waves, we evaluate the wave growth as a function of position and wavenumber for a specified proton injection energy spectrum, $\mathrm{d}N/\mathrm{d}E$. The wave growth is found to be proportional to $vp\,\mathrm{d}N/\mathrm{d}E$, where v and p are the particle speed and momentum, and to the local Alfvén speed V_A. Thus, maximum wave growth is achieved at the location of maximum V_A (at a few solar radii), and the minimum value of $\mathrm{d}N/\mathrm{d}E$ required for the wave growth to exceed unity there is a few times $10^{32}/vp$ protons per unit solid angle (in coordinate space) at the solar surface. If $\mathrm{d}N/\mathrm{d}E$ is below this value, test-particle theory is a valid description of particle transport and acceleration. The value is not exceeded (above 1 MeV energies) in small gradual SEP events having peak 1-MeV proton intensities below $\sim$10 protons (cm^2 sr s MeV)$^{-1}$ at 1 AU. The spatial and momentum dependence of the wave growth can also be used to estimate the maximum emission strength of a moving proton source in the interplanetary medium. For a strong source moving through the solar wind at constant super-Alfvénic speed, the number of escaping particles per unit time and flux-tube cross section is approximately constant in time, predicting a plateau-type time–intensity profile observed ahead of the source. The model reproduces observations of streaming-limited intensities at energies around 10 MeV and explains the double peaked injection profiles observed in large SEP events.

Key words. instabilities – Sun: particle emission – turbulence

1. Introduction

After over half a century of their discovery (Forbush 1946), the origin of solar energetic particles (SEPs) is still under discussion. The most widely accepted theory of particle acceleration in large SEP events is diffusive shock acceleration (Bell 1978) in CME-driven shock waves (see, e.g., Reames 1999 for a review). Substantial energy gains by the diffusive mechanism are possible only if the diffusion length of the energetic particles close to the shock is much less than the macroscopic length scale (of the order of the heliocentric distance, r) of the system (Vainio et al. 2000). To allow the particles to escape ahead of the shock to the ambient medium, where they are detected just tens of minutes after the onset of the eruption at the Sun, the turbulent region ahead of the shock has to be very limited in scale (Vainio et al. 2000; Zank et al. 2000; Li et al. 2003). Such a model of the magnetic fluctuations is provided by Alfvén waves that are generated ahead of the shock by the streaming accelerated particles themselves (e.g., Bell 1978). The theory of self-generated Alfvén waves has been successfully applied to explain, e.g., particle acceleration in interplanetary (IP) shock waves near 1 AU (Lee 1983; Kennel et al. 1986), as well as "streaming limited" intensities (Ng & Reames 1994; Reames & Ng 1998) and systematic time variations of ion abundances (Tylka et al. 1999; Ng et al. 1999) observed in large SEP events.

While it is generally believed that self-generated waves provide the turbulence necessary for particle acceleration in large SEP events, a dynamical model including all aspects of the problem in a time dependent manner still waits to be constructed. Models of particle acceleration (Bell 1978; Lee 1983) are based on a 1-dimensional quasi-stationary state around the shock, while dynamical transport models (Ng & Reames 1994; Ng et al. 1999) do not treat the acceleration consistently, but rely on ad-hoc particle injection at the shock. The recent study by Li et al. (2003) (see also Zank et al. 2000) uses a quasi-stationary acceleration model to describe the particle distribution function at the shock combined with an ad-hoc escape condition to describe the proton injection from the shock. Their study, however, neglects wave generation by the escaping particles. Furthermore, these authors did not model particle acceleration below 0.1 AU. Thus, basing on the theoretical work done so far, it is still not very easy to judge the role of self-generated waves in SEP events of various strength.

In this paper we apply a simplified model of the streaming instability to derive analytical expressions that may be used

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to evaluate the role of self-generated waves in SEP events of various strength. To illustrate the use of the model, we study events analyzed previously by test-particle methods. Validity of the test-particle theories and the role of self-generated waves in these events is discussed, and a simple model of proton escape from strong IP shocks is developed.

2. Model

Consider energetic protons scattering off outward-propagating Alfvén waves in the IP medium. In each scattering, the proton's pitch-angle cosine, μ, changes by amount $\Delta\mu$, while the proton momentum p, both measured in the wave frame, stays constant. This scattering changes the plasma-frame proton energy by $V_A p\Delta\mu$, where V_A is the Alfvén speed. Thus, conservation of total plasma-frame energy of the wave and of the proton leads to a change of the wave energy by $\Delta E_w = -V_A p\Delta\mu$. Summing up the contributions from all protons at a given position gives the rate of change of the wave energy density as

$$\frac{dU_w}{dt} = -\int d^3p\, V_A p \frac{\langle\Delta\mu\rangle}{\Delta t}\, f(\boldsymbol{r}, \boldsymbol{p}, t), \tag{1}$$

where f is the proton distribution function. The Fokker–Planck coefficient $\langle\Delta\mu\rangle/\Delta t = \partial D_{\mu\mu}/\partial\mu$, where $D_{\mu\mu} = \frac{1}{2}\langle(\Delta\mu)^2\rangle/\Delta t$ is the particle's pitch-angle diffusion coefficient (Jokipii 1966). Assuming, for simplicity, that the waves propagate parallel to the mean magnetic field, we can write the wave-energy density as $U_w = \int dk\, W(k)$, where $W(\boldsymbol{r}, k, t)\, dk$ is the energy density of waves with wavenumber in the range from k to $k + dk$. This can be related to the pitch-angle diffusion coefficient using the quasilinear theory (Jokipii 1966):

$$D_{\mu\mu} = \frac{\pi}{4}\omega_c(1-\mu^2)\frac{|k_r|W(k_r)}{U_B}, \tag{2}$$

where $\omega_c = \omega_{cp}/\gamma$, $\omega_{cp} = eB/m_p c$, is the non-relativistic proton cyclotron frequency, e is the elementary charge, m_p is the proton mass, c is the speed of light, γ is the proton Lorentz factor, $U_B = B^2/8\pi$ is the magnetic field energy density, $k_r = -\omega_c/(v\mu)$ is the resonant wavenumber, and v is the particle speed in wave frame. Thus, after a small amount of algebra, the wave growth rate can be obtained as

$$\begin{aligned}\Gamma(k) &= \frac{1}{W}\frac{dW}{dt} \\ &= \frac{\pi}{2}\frac{\omega_{cp}}{nV_A}\int d^3p\, v\,(1-\mu^2)|k|\,\delta(k+\omega_c/v\mu)\frac{\partial f}{\partial\mu}\end{aligned} \tag{3}$$

where n is the proton density of the background medium and $m_p n$ is taken as the mass density.

Aiming at analytical estimates of the wave growth, we will not use the exact equation of the growth rate. Instead, we simplify the quasilinear resonance condition by neglecting its μ dependence (e.g., Skilling 1975; Bell 1978; Lee 1983). Thus, we replace $\delta(k+\Omega/v\mu)$ by $\frac{1}{2}\delta(|k|-\omega_c/v)$. This approximation breaks down for circularly polarized waves if the odd part of $\partial f/\partial\mu$ is larger than the even part, leading to a significant asymmetry in waves generated at positive and negative wavenumbers. However, if the wave has a finite but small angle of propagation, implying linear polarization, the resonance condition becomes $k_r = \pm\omega_c/(v\mu)$ and the positive and negative wavenumbers are similarly excited (e.g., Skilling 1975). Thus, we regard the approximation to be a useful order-of-magnitude estimate of the wave growth rate, when applied to the calculation of time-integrated wave growth. For fully time-dependent modeling, this approximation is obviously too simple: Waves at any k are first amplified by fast protons scattered to small μ, then by slower protons that survive scattering at large μ. The process involves protons over a range of energies and pitch angles, which should be taken into account in rigorous dynamical modeling.

Using partial integration in μ we get

$$\Gamma(k) = \frac{\pi}{2}\frac{\omega_{cp}}{nV_A}\int d^3p\, v\mu\,|k|\,\delta(|k|-\omega_c/v)\, f. \tag{4}$$

We then take the resonant momentum, $p = m_p\omega_{cp}/|k|$, as the independent variable to get

$$\Gamma(\boldsymbol{r}, p, t) = \frac{\pi}{2}\,\omega_{cp}\,\frac{p\,S(\boldsymbol{r}, p, t)}{nV_A}, \tag{5}$$

where $S(\boldsymbol{r}, p, t) = 2\pi\int_{-1}^{+1} d\mu\, v\mu\, p^2 f$ is the streaming of protons, i.e., the net number of protons crossing a point in space per unit area (perpendicular to the magnetic field), unit momentum and unit time, at (wave-frame) momentum p. From now on, we will neglect the small difference between the momenta measured in the wave frame and in the fixed frame, and regard $S(p)$ to be the streaming measured in the fixed frame. This, of course, requires that protons propagate much faster than the waves through the IP medium.

Neglecting wave propagation (like, e.g., Ng & Reames 1994), we can write the evolution equation of the Alfvén waves as

$$\frac{\partial W}{\partial t} = \Gamma W \quad\Rightarrow\quad W = W_0\exp\left\{\int_0^t dt'\,\Gamma(t')\right\}. \tag{6}$$

This gives the conditions for significant wave growth as

$$1 < \int_0^t dt'\,\Gamma(t') = \frac{\pi}{2}\,\omega_{cp}\frac{p\,F(\boldsymbol{r}, p, t)}{nV_A}, \tag{7}$$

where

$$F(\boldsymbol{r}, p, t) = \int_0^t dt'\, S(\boldsymbol{r}, p, t') \tag{8}$$

is the time-integrated net proton flux. Let us, then, denote the cross sectional area of the flux tube by $A(r)$. Note that $A \propto B^{-1}$. Assuming that perpendicular diffusion is negligible, protons stay in a single flux tube. Thus, the proton position can be denoted by r. Neglecting adiabatic energy losses during IP transport then makes $\mathcal{F}(p) = A(r)F(r, p, \infty)$ independent of r at all distances outward from the source. Assuming, that no particles are absorbed at the Sun, all particles eventually escape to $r \to \infty$. Thus, $\mathcal{F}$ is given by the number of protons per unit momentum injected to the flux tube at momentum p, i.e.,

$$\mathcal{F}(p) = \frac{dN}{dp}. \tag{9}$$

The equation for the wave growth integrated over the whole SEP event becomes

$$\int_0^\infty \mathrm{d}t\,\Gamma(r,p,t) = \frac{\pi}{2}\omega_{\mathrm{cp}}(r)\frac{p\mathcal{F}(p)}{n(r)A(r)V_{\mathrm{A}}(r)} = \frac{\pi}{2}\omega_{\mathrm{cp}}(r_0)\frac{p\mathcal{F}(p)}{n(r_0)A(r_0)V_{\mathrm{A}}(r_0)}\frac{V_{\mathrm{A}}(r)}{V_{\mathrm{A}}(r_0)}, \tag{10}$$

where r_0 is an arbitrary reference distance and where we have made use of the fact that the factor $\omega_{\mathrm{cp}}/nV_{\mathrm{A}} = (4\pi e/c)(V_{\mathrm{A}}/B) \propto AV_{\mathrm{A}}$. This equation can be used to analyze, whether wave growth is important in an event at different distances from the source. Note that in case the focusing length, $L = A/(\partial A/\partial r)$, is much shorter than the scattering mean free path of the protons, our approximation neglecting the μ-dependence of the resonance condition may become invalid. This means that Eq. (10) should only be taken as qualitative estimate close to the Sun. Note also, that since it takes a longer time at larger distances for the time-integrated net flux to saturate, Eq. (7) actually predicts a steeper dependence of the wave growth than proportionality to V_{A} predicted by Eq. (10).

3. Results and discussion

3.1. Wave growth between the Sun and 1 AU

First, we evaluate the SEP source spectra that can lead to significant wave growth in solar corona and IP medium inside 1 AU. The cross sectional area of the flux tube can be written as

$$A(r) = r_\odot^2\,\mathrm{d}\Omega_\odot\frac{B_\odot}{B(r)}, \tag{11}$$

where $B(r)$ is the magnetic field magnitude, $r_\odot$ is the solar radius, $B_\odot = B(r_\odot)$, and $\mathrm{d}\Omega_\odot$ is the (differential) solid angle of the flux tube at the solar surface. The momentum spectrum can be written as

$$\mathcal{F}(p) = \mathrm{d}\Omega_\odot v\frac{\mathrm{d}^2N}{\mathrm{d}E\,\mathrm{d}\Omega_\odot}, \tag{12}$$

where $\mathrm{d}^2N/(\mathrm{d}E\,\mathrm{d}\Omega_\odot)$ is the energy spectrum of protons per unit solid angle at the solar surface (in coordinate space) injected into the flux tube, which can be deconvolved from observations of SEP intensities and anisotropies (e.g., Torsti et al. 1996; Anttila et al. 1998; Anttila & Sahla 2000; Laitinen et al. 2000). Plugging these expressions in Eq. (10), we get

$$\int_0^\infty \mathrm{d}t\,\Gamma(r,p,t) = vp\frac{\mathrm{d}^2N}{\mathrm{d}E\,\mathrm{d}\Omega_\odot}\frac{\pi\,\omega_{\mathrm{cp}\odot}}{2\,r_\odot^2 n_\odot V_{\mathrm{A}\odot}}\frac{V_{\mathrm{A}}(r)}{V_{\mathrm{A}\odot}} = 10^{-33}\,vp\frac{\mathrm{d}^2N}{\mathrm{d}E\,\mathrm{d}\Omega_\odot}\sqrt{\frac{2\times10^8\,\mathrm{cm}^{-3}}{n_\odot}}\frac{V_{\mathrm{A}}(r)}{V_{\mathrm{A}\odot}}. \tag{13}$$

We note, first, that since $V_{\mathrm{A}\odot}/V_{\mathrm{A}}(1\,\mathrm{AU}) \sim 10$ for typical coronal and IP parameters, the source spectrum has to exceed $\sim 5\times10^{33}/E$ particles per steradian at the solar surface, before any substantial wave growth at 1 AU can be expected. Furthermore, wave growth peaks at the distance of maximum Alfvén speed, $V_{\mathrm{A,max}}$. This is typically located at $r \sim 2$–$4\,r_\odot$ and has a value of a few times $V_{\mathrm{A}\odot}$, when evaluated using the magnetic fields and densities empirically determined for the equatorial background corona (see, e.g., Bird & Edenhofer 1990).

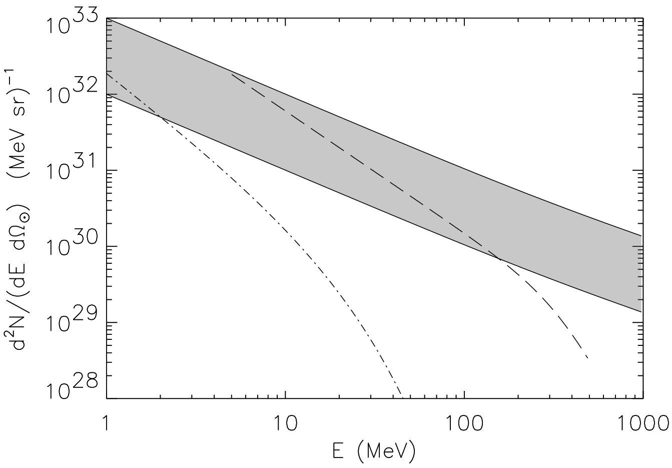

Fig. 1. The estimated critical source spectrum leading to wave growth in solar corona (*shaded area bounded by the solid curves*). The source of 7 April 1997 and 12 May 1997 SEP events (*dot-dashed curve*) and the prompt-component spectrum of 24 May 1990 SEP event (*dashed curve*) are plotted for comparison.

Thus, wave growth is weakening in the IP space as a function of radial distance, and even if we don't observe it locally at 1 AU, there might be significant wave growth in solar corona enabling efficient particle acceleration in coronal shocks. In fact, since it leads to turbulent particle trapping, strong wave growth should be observable locally only when a source of the energetic particles, e.g., an IP shock wave, is close to the observer.

Our expression allows us to write down an upper limit for the injection spectrum for events permitting the use of test-particle transport theories all the way from the Sun to 1 AU. This becomes

$$\frac{\mathrm{d}^2N}{\mathrm{d}E\,\mathrm{d}\Omega_\odot} \lesssim \frac{10^{33}}{vp}\sqrt{\frac{n_\odot}{2\times10^8\,\mathrm{cm}^{-3}}}\frac{V_{\mathrm{A}\odot}}{V_{\mathrm{A,max}}}, \tag{14}$$

amounting to a few times $10^{32}/vp$ for typically adopted parameters. (Note, however, that in case of active regions, $n_\odot$ can be two orders of magnitude larger than the adopted value). A similar analytical estimate for negligible wave growth was shown to agree with time-dependent simulations by Vainio & Kocharov (2001) in case of proton transport through self-generated waves in flaring coronal loops.

Let us investigate whether typical small gradual events, like the 7 April and 12 May 1997, events analyzed by Anttila & Sahla (2000), are below the wave growth threshold. The time-integrated injection spectrum in both events can be given approximately by $E\,\mathrm{d}^2N/(\mathrm{d}E\,\mathrm{d}\Omega_\odot) \approx 2\times10^{32}(\mathrm{MeV}/E)^{0.8}\exp(-E/15\,\mathrm{MeV})$ (see Fig. 1). According to Anttila & Sahla (2000), most part of the injection occurs when the source (the CME) is above probable heights of the Alfvén speed maximum, so only a fraction of this spectrum is available to generate waves in the low coronal regions near the Alfvén speed maximum. Thus, we conclude that test particle approximation is justified for these events at energies $E > 1$ MeV. Both events had maximum intensities at 1.6–3 MeV energy channel of ~ 3 protons $(\mathrm{cm}^2\,\mathrm{sr\ s\ MeV})^{-1}$. We, therefore, estimate as a rule of thumb that particle transport in SEP events having peak intensities below ~ 10 protons $(\mathrm{cm}^2\,\mathrm{sr\ s\ MeV})^{-1}$ at 1 MeV can

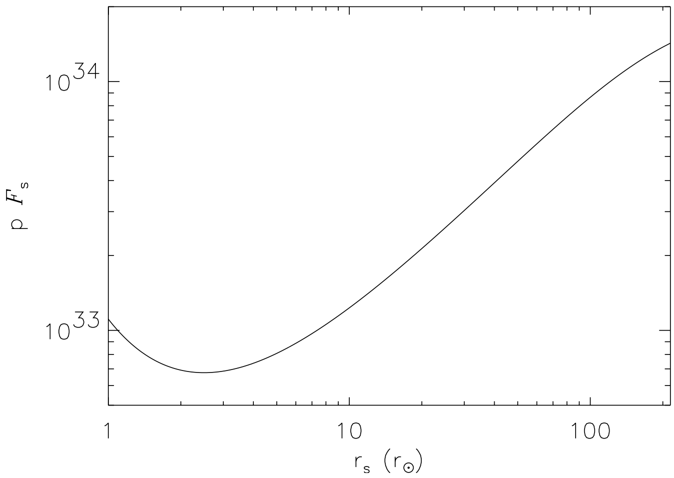

Fig. 2. The time integrated proton flux at the source boundary as a function of source position.

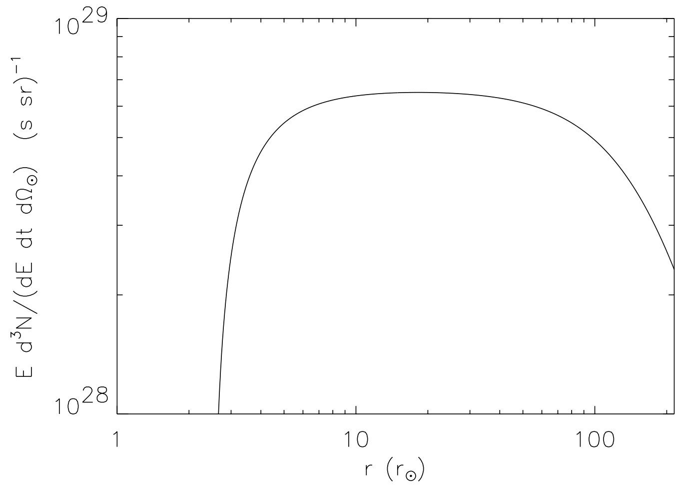

Fig. 3. The numerically evaluated injection rate from a strong source for non-relativistic protons for typical coronal magnetic fields and electron densities.

be analyzed using test-particle methods. On the other hand this also implies that one can not expect shock acceleration with self-generated waves as the upstream scattering centers to produce the SEPs in such small events.

3.2. SEP escape from a strong, super-Alfvénic source

Let us, next, use our model to derive the spectrum of escaping protons from a strong, super-Alfvénicly propagating source, like a fast CME-driven shock wave. Let us denote the position of the leading edge of the source by $r_s(t)$ and take the radial speed of the moving source $V_s = \dot{r}_s > 0$. Thus, $\mathcal{F}_s(r_s, p) = A(r_s)F(r_s, p, t)$ gives net the number of particles per unit momentum that have crossed the flux-tube cross sectional area at $r = r_s(t)$ at times $0 < t' < t$. We calculate the wave growth at the source position as

$$\alpha(r_s, p) \equiv \int_0^t dt'\, \Gamma(r_s, p, t') = \left.\frac{\pi\omega_{cp}\, p}{2nAV_A}\right|_{r=r_s} \mathcal{F}_s(r_s, p). \tag{15}$$

We can now argue that α has to have a value of a few: inside the source, particles are trapped only if α is large and outside the source they can escape only if α is not large. Taking, thus, α to be a constant of unit order enables us to give $\mathcal{F}_s$ in form

$$\mathcal{F}_s(r_s, p) = \left.\frac{2\alpha\, nAV_A}{\pi\omega_{cp}\, p}\right|_{r=r_s} = \frac{2\alpha\, n_0 A_0 V_{A0}}{\pi\omega_{cp0}\, p} \frac{V_{A0}}{V_A(r_s)}, \tag{16}$$

where quantities subscripted with 0 are taken at an arbitrary reference point, $r = r_0$. We evaluate $p\mathcal{F}_s$ using the following model of corona (Model A): a magnetic field of $B = B_r\sqrt{1 + (r/\mathrm{AU})^2}$ with $B_r = [1.3\,(r_\odot/r)^2 + 1.7\,(r_\odot/r)^3]$ G, and the equatorial density profile of Sittler & Guhathakurta (1999). (Note that the parameter a_1 in their Table 1 should be multiplied with 10^8 to get densities in cgs units.) The result is plotted in Fig. 2 for $r_0 = r_\odot$, $A_0 = r_\odot^2$, and $\alpha = 1$.

In the left-most part of the curve, $\mathcal{F}_s$ is decreasing as a function of position, which indicates that particles are trapped to the source more efficiently as the source moves outward. When, however, the source is moving toward a decreasing V_A, $\mathcal{F}_s$ increases as a function of radius. Thus, trapping becomes less efficient and this allows particles to stream out from the source until trapping is resumed. The momentum spectrum of particles escaping between times t and $t + \Delta t$ is given by $\Delta\mathcal{F}_s = \mathcal{F}_s(r_s + V_s\Delta t) - \mathcal{F}_s(r_s)$. The number of particles in a given flux tube per unit momentum and unit time escaping to the ambient medium is, therefore,

$$\begin{aligned} Q &\equiv \frac{d^2N}{dp\, dt} = \lim_{\Delta t\to 0} \frac{\Delta\mathcal{F}_s}{\Delta t} = V_s \frac{\partial\mathcal{F}_s}{\partial r_s} \\ &= \frac{2\alpha\, n_0 A_0 V_{A0}^2}{\pi\omega_{cp0}\, p} \left(V_s \frac{\partial V_A^{-1}}{\partial r}\right)_{r=r_s}. \end{aligned} \tag{17}$$

This, of course, only applies if the derivative is positive.

Let us then consider a simplified model of the inner heliosphere (Model B) with a radial magnetic field ($B \propto r^{-2}$) and a constant solar wind speed (yielding $n \propto r^{-2}$). Thus, $V_A = V_{A0} r_0/r$, $r_0 = 1$ AU, and $A_0 = r_0^2\, d\Omega_\odot$. This gives,

$$\begin{aligned} \frac{d^3N}{dE\, dt\, d\Omega_\odot} &= \frac{Q}{v\, d\Omega_\odot} = \frac{2\,\alpha\, n_0\, r_0\, V_s\, V_{A0}}{\pi\omega_{cp0}\, vp} \\ &= \frac{2\alpha\, V_{s3}\, n_1^{1/2}}{vp}\, 3.4\, 10^{28}\ \mathrm{sr}^{-1}\ \mathrm{s}^{-1}, \end{aligned} \tag{18}$$

where $V_s = V_{s3}\, 10^3$ km s^{-1} and $n_0 = n_1\, 10^1$ cm^{-3} have been used. This injection is a constant, consistent with the plateau-type intensity profiles observed in connection with the streaming-limited intensities of the large SEP events (Reames & Ng 1998; Reames 1999).

Choosing $r_0 = r_\odot$ and $A_0 = r_\odot^2 d\Omega_\odot$, we can write

$$\frac{d^2N}{dE\, dt\, d\Omega_\odot} = \frac{2\alpha\, n_\odot r_\odot^2 V_{A\odot}^2}{\pi\omega_{cp\odot}\, pv} \left(V_s \frac{\partial V_A^{-1}}{\partial r}\right)_{r=r_s}. \tag{19}$$

We evaluate this numerically using Model A. The result is plotted in Fig. 3 for non-relativistic (i.e., $E = \frac{1}{2}vp$) protons with $V_{s3} = \alpha = 1$. This injection tends toward zero as $r \to \infty$ because in a spiral field, V_A tends toward a constant value at the same limit. Note that since we neglected wave propagation, our model is applicable only to source speeds clearly larger than the sum of the ambient solar-wind and Alfvén speeds (i.e., $V_3 > 1$). For the same reason we regard the model valid only for the first, say, 10 hours of the event, so the modeled escape beyond about 0.3 AU with decreasing values may not be reliable.

Let us compare the obtained values with the 19 October 1989 event analyzed earlier by Anttila et al. (1998) and Anttila & Sahla (2000). This event was one of the most intensive SEP events ever observed, and was related to a CME-driven IP shock wave with a speed of ∼2000 km s^{-1}, so the assumption of a strong super-Alfvénic source seems reasonable. The 10-MeV injection profile was approximately constant, when the shock was between 0.04 and 1 AU, and had a value of $3–4 \times 10^{28}$ protons (sr s MeV)$^{-1}$. Comparing the value with our Model B estimate indicates, that $\alpha n_1^{1/2} \approx 5$ would explain the emission from the shock at 10 MeV. The injection spectrum was, however, slightly softer than our estimate, approximately $\propto E^{-1.6}$ between 10 and 100 MeV. We speculate that this could be, e.g., because of the inability of our simple model to take proper account of the effects of scattering near $\mu = 0$, which can not be due to outward-propagating Alfvén waves in quasi-linear theory (see, e.g., Vainio 2000). If low-energy particles need to generate more waves to become trapped than our prediction and/or if the high-energy protons can become trapped by waves generated by low-energy protons, we could expect softening of the escaping spectrum. Similarly, softening could result from a damping process that would be stronger at higher wavenumbers. Or, if the source spectrum is softer than $E^{-1.6}$, more particles at low energies could be able to diffuse outward despite the wave growth. Thus, the trapping region could even be more extended at low energies, explaining the softness of the spectrum (see Fig. 2).

3.3. Double peaked injection

As a last topic, we discuss the double peaked intensities observed in many large SEP events, e.g., 19 October 1989 (Anttila et al. 1998) and 24 May 1990 (Torsti et al. 1996). In these events, and many others, a prompt injection of protons, starting rapidly after the onset of the eruption, is followed by a larger delayed injection that peaks when the leading edge of the associated CME is beyond 5 $r_\odot$. We can use the typical Alfvén speed profile to understand this phenomenon. The prompt component could consist of particles that are accelerated near the Sun and streaming away from the source before the waves have grown appreciably to stop their propagation. The delayed component injection could only start once its source, the CME shock, reaches distances where V_A decreases, i.e., $r \gtrsim 3\, r_\odot$. This hypothesis can be tested by comparing the deduced injection component of the prompt component to our critical spectrum, Eq. (14). The prompt-component spectrum of May 24, 1990, SEP event was fitted by Torsti et al. (1996) as

$$\frac{d^2N}{dE\, d\Omega_\odot} = 2.4 \times 10^{33} \frac{(\mathrm{MeV}/E)^{1.6}}{1 + (E/360\,\mathrm{MeV})^3}\ \mathrm{sr}^{-1}\,\mathrm{MeV}^{-1} \qquad (20)$$

in the range 5–500 MeV. While this spectrum is again somewhat softer than our prediction ($\propto E^{-1}$) the values at 10–100 MeV agrees very well with the theory (see Fig. 1). Finally, in this event, the delayed injection started relatively sharply, when the CME shock was around 20 $r_\odot$, which is only marginally consistent with our simple model (see Fig. 3).

4. Conclusions

We have presented a simple analytical model of the evolution of Alfvén waves in the solar corona and IP medium subjected to wave growth due to streaming SEPs. The model is not aimed at replacing detailed numerical computations. Instead, we wanted to use it for obtaining qualitative understanding of the wave growth and for the analysis of observational data. We demonstrated that the time-integrated wave growth rate is proportional to the Alfvén speed and to the source spectrum evaluated at the resonant wave number, $(vp\, dN/dE)_{p=m_p\omega_{cp}/|k|}$, see Eq. (13). This result allowed us to reach the following conclusions

1. SEP events require a proton spectrum of a few times $10^{32}/E$ per steradian at the solar surface to be emitted to IP medium before wave growth due to streaming instability can be important.
2. Small SEP events with peak intensities below ∼10 protons (cm^2 sr s MeV)$^{-1}$ at 1 MeV and 1 AU can usually be analyzed using test particle methods. Shock acceleration in such small events is unlikely to be significantly intensified by wave growth.
3. Proton energy spectrum emitted from a strong, fast-moving source is predicted to be temporally constant and have a spectral index of −1. While the slope is a bit too hard to explain typical observations, the value of the source spectrum at 10 MeV is consistent with the plateau-type intensities observed in the largest SEP events, like that on 19 October 1989.
4. If coronal Alfvén speed has a maximum at a few solar radii above the surface (as inferred from typical empirical models of equatorial coronal magnetic fields and densities), the model predicts a double-peaked proton injection: a prompt component escaping before the trapping due to wave growth, and a delayed component starting, when the fast-moving source (i.e., the CME-driven shock) has propagated beyond the Alfvén speed maximum. The prompt-component spectrum should be below $10^{33}/E$ protons per steradian at the solar surface, consistent with the event of 24 May 1990 at energies above 5 MeV.

However, since the model is based on a modified wave-particle resonance neglecting the μ-dependence, the quantitative aspects of our study are still tentative. Work on a more rigourous treatment is in progress.

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