

# New results on the spectral index–flux density relation from the WENSS/NVSS catalogs<sup>★</sup>

X. Zhang<sup>1,2</sup>, W. Reich<sup>2</sup>, P. Reich<sup>2</sup>, and R. Wielebinski<sup>2</sup>

<sup>1</sup> National Astronomical Observatories, CAS, Beijing 100012, PR China

<sup>2</sup> Max-Planck-Institut für Radioastronomie, Auf dem Hügel 69, 53121 Bonn, Germany

Received 30 July 2002 / Accepted 24 March 2003

**Abstract.** We present new statistical results on the spectral index–flux density relation for large samples of radio sources using archival data of the most sensitive surveys, such as WENSS and NVSS. Instrumental selection effects and the completeness of the catalogs used in this study are discussed. Our main results are based on the spectral indices calculated for about 185 800 sources from the WENSS (327 MHz) and the NVSS (1.4 GHz) catalogs and are summarized as follows: (1) The median spectral index increases from  $\alpha_{\text{med}} \sim -0.9$  to  $\alpha_{\text{med}} \sim -0.8$  ( $S_{\nu} \propto \nu^{\alpha}$ ) for  $S_{327}$  flux densities decreasing from 0.1 Jy down to 23 mJy. The median spectral indices nearly show no variation within the error bars in the flux density range above 100 mJy up to several Jy. The median spectral index slightly increases again for  $S_{327}$  above several Jy. The new results confirm published models of the radio luminosity function (RLF) for sources with  $S_{327} > 0.1$  Jy and give constraints to the models for sources of  $0.023 \text{ Jy} < S_{327} < 0.1 \text{ Jy}$ , respectively. (2) A dependence of the fractions of ultra-steep-spectrum sources (USS,  $-1.5 \leq \alpha < -1.0$ ), steep-spectrum sources (SSS,  $-1.0 \leq \alpha < -0.5$ ) and flat-spectrum sources (FSS,  $-0.5 \leq \alpha \leq 0.0$ ) is partly responsible for the spectral flattening. Another contribution to the spectral flattening comes from the variation of  $\alpha_{\text{med}}$  of steep-spectrum sources ( $\alpha < -0.5$ ) themselves which increases with decreasing flux densities. (3) The spectral flattening for faint sources (down to  $S_{327} \sim 20$  mJy) with steep spectra ( $\alpha < -0.5$ ) suggests that  $\alpha_{\text{med}}$  is correlated with luminosity rather than redshift according to the source evolution model of Condon (1984).

**Key words.** methods: statistical – radio continuum: galaxies – galaxies: evolution

## 1. Introduction

Models of the radio luminosity function (RLF) published in the past years describe the cosmological evolution of radio source samples. So far most RLF-models attempt to fit observed source counts, luminosity or redshift distributions of source samples, which, however, are restricted to relatively strong sources (Peacock & Gull 1981; Wall et al. 1980, 1981; Subrahmanya & Kapahi 1983; Condon 1984; Dunlop & Peacock 1990; Jackson & Wall 1999). In general the  $\alpha_{\text{med}}-S$  relation predicted by RLF-models were just partially confirmed by available data. However, the source samples used so far were rather limited in size and sensitivity. Differences between the observed and the predicted  $\alpha_{\text{med}}-S$  relation provide constraints to RLF-models, which are of particular interest for the faint sources' properties. This needs observed  $\alpha_{\text{med}}-S$  relations of high quality for a wide range of flux densities in order to confirm RLF-models or set constraints for improvements.

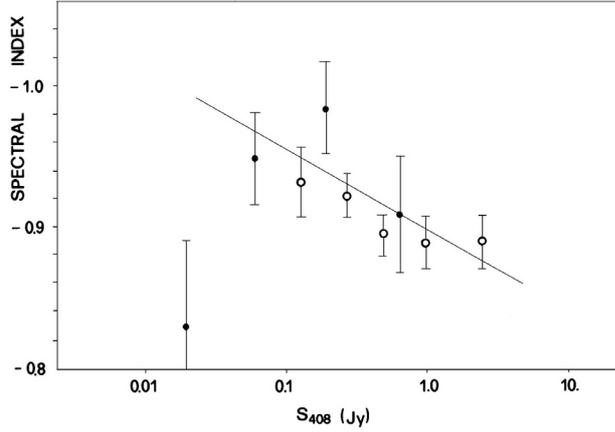
Several investigations on the  $\alpha_{\text{med}}-S$  relation came to rather different conclusions based on small or compound samples. Vigotti et al. (1989) reported that the median spectral index

decreases from  $\alpha_{\text{med}} \sim -0.90$  to  $\alpha_{\text{med}} \sim -0.96$  for flux densities ranging from 1 Jy to 0.1 Jy at 408 MHz (Fig. 1). They used a sample of 1103 sources selected from the B3 catalog at 408 MHz (Ficarra et al. 1985) and calculated spectral indices using 1465 MHz data measured with the VLA. Steppe & Gopal-Krishna (1984) reported a contrary result for a radio source sample in the same flux density range at 408 MHz:  $\alpha_{\text{med}}$  was found to be about  $-0.9$  at the 1 Jy and  $-0.75$  at the 0.1 Jy flux density level (Fig. 2). Their sample of 1009 sources was taken from several catalogs including 5C12 (Benn et al. 1982), B2 (Grueff & Vigotti 1979), MC1 (Davies et al. 1973), MC2 and MC3 (Large et al. 1981), All-sky Survey (Robertson 1973). Kapahi & Kulkarni (1986), Kulkarni & Mantovani (1985, 1985) and Kulkarni et al. (1990) claimed that the median spectral index is constant at  $\alpha_{\text{med}} \sim -0.9$  in the same flux density range from 1 Jy to  $\sim 0.1$  Jy. A constant  $\alpha_{\text{med}}$  differs from the results of Vigotti et al. (1989) and Steppe & Gopal-Krishna (1984), as mentioned above.

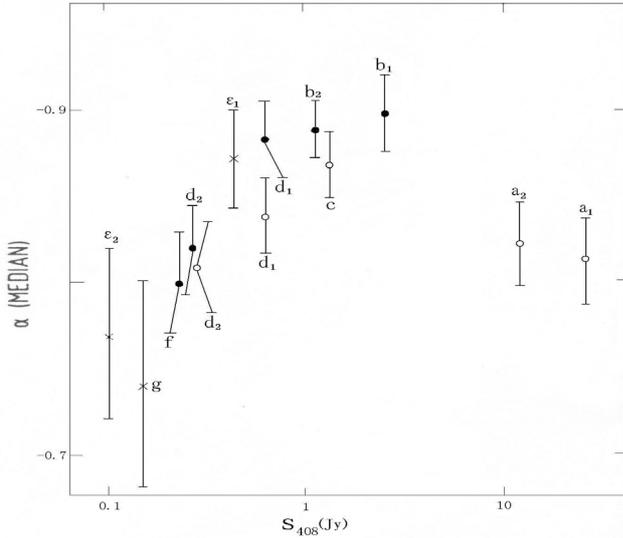
A few radio source surveys with large sky coverage and high sensitivities at different frequencies have recently been published providing a new basis to study the spectral index–flux density relation, especially for sources with  $S_{327} < 0.1$  Jy. In Sect. 3 we give the results of the spectral index–flux density relation using WENSS (Rengelink et al. 1997) and NVSS

Send offprint requests to: X. Zhang, e-mail: zxz@ns.bao.ac.cn

<sup>★</sup> Supported by the National Science Foundation of China under Grant No. 10173015 and the Partner Group of MPIfR at NAOC.



**Fig. 1.** Result of Vigotti et al. (1989) on the relation between flux density and spectral index for frequencies of 408 MHz and 1465 MHz. The data are from B3 (open circles, Ficarra et al. 1985) and Benn et al. (1988) (filled circles).



**Fig. 2.** The result of Steppe & Gopal-Krishna (1984) of the flux density–spectral index relation for frequency of 408 MHz (except sample g which is defined at 611 MHz) and a higher frequency which is 1.4, 2.7 or 5.0 GHz as indicated by crosses, open circles and dots, respectively.

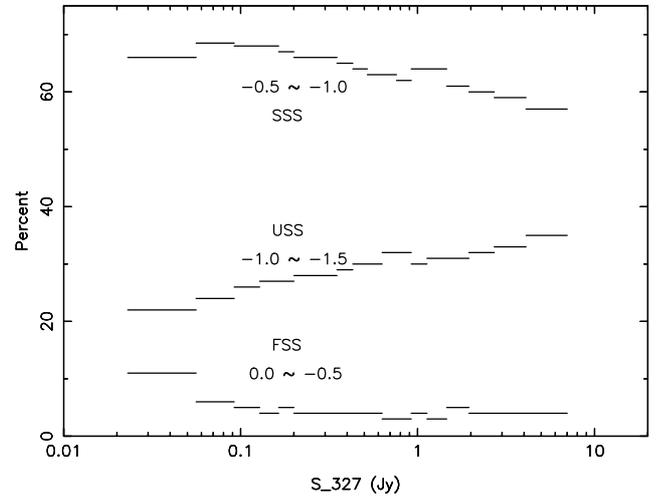
(Condon et al. 1998) data. Investigations based on the other large catalogs are mainly used for comparison which will be given elsewhere (Zhang et al. 2003). Selection effects and completeness of radio source samples for both statistics and observations are discussed in Sect. 2. The results of an analysis of the WENSS and NVSS data are presented in Sect. 3. In Sect. 4 a discussion and a comparison of the new results with luminosity evolution models are made.

## 2. The radio source samples

Several radio source surveys with large sky coverages became publicly available. The source catalog “NVSS29” used in this study was selected from the catalog of NVSS carried out with the VLA (Condon et al. 1998) for  $\delta \geq 29^\circ$ . This declination

**Table 1.** The fractions of radio sources in different  $\alpha_{327}^{1400}$  ranks and flux density bins at 327 MHz. The radio sources are from the WENSS and NVSS catalogs.

$S_{327}$ (Jy)	$-2.0 \leq \alpha$ $< -1.5$	$-1.5 \leq \alpha$ $< -1.0$	$-1.0 \leq \alpha$ $< -0.5$	$-0.5 \leq \alpha$ $\leq 0.0$
.038 <sup>.023</sup> <sub>.056</sub>	.02	.22	.65	.11
.074 <sup>.056</sup> <sub>.092</sub>	.01	.24	.68	.06
.110 <sup>.092</sup> <sub>.128</sub>	.01	.26	.68	.05
.146 <sup>.128</sup> <sub>.164</sub>	.01	.27	.68	.04
.182 <sup>.164</sup> <sub>.200</sub>	.01	.27	.67	.05
0.22 <sup>.200</sup> <sub>.240</sub>	.01	.28	.66	.04
0.27 <sup>.240</sup> <sub>.290</sub>	.01	.28	.66	.04
0.32 <sup>.290</sup> <sub>.350</sub>	.01	.28	.66	.04
0.39 <sup>.350</sup> <sub>.430</sub>	.01	.29	.65	.04
0.48 <sup>.430</sup> <sub>.520</sub>	.01	.30	.64	.04
0.58 <sup>.520</sup> <sub>.630</sub>	.01	.30	.64	.04
0.70 <sup>.630</sup> <sub>.760</sub>	.01	.32	.63	.03
0.84 <sup>.760</sup> <sub>.920</sub>	.01	.32	.63	.03
1.03 <sup>.920</sup> <sub>1.13</sub>	.01	.30	.64	.04
1.30 <sup>1.13</sup> <sub>1.46</sub>	.01	.31	.64	.03
1.71 <sup>1.46</sup> <sub>1.95</sub>	.01	.31	.61	.05
2.33 <sup>1.95</sup> <sub>2.71</sub>	.01	.32	.60	.04
3.41 <sup>2.71</sup> <sub>4.10</sub>	.01	.33	.59	.04
5.55 <sup>4.10</sup> <sub>7.00</sub>	.03	.35	.57	.04



**Fig. 3.** The variation of the source fraction for FSS sources (bottom), SSS sources (top) and USS sources (middle) at 327 MHz.

limit matches the most important catalog at low frequencies, the 327 MHz WENSS catalog (Rengelink et al. 1997), which was carried out with the Westerbork Synthesis Radio Telescope and covers the sky for  $\delta \geq 30^\circ$ .

To ensure statistical completeness a flux limitation of the statistical study (FLSS) is defined first. Most radio sources selected at low frequencies such as 327 MHz have spectral indices larger than  $\alpha = -1.5$ . This holds for a large flux density range. Figure 3 shows the fractional variation of radio sources

whose spectral indices fall into the ranges of  $-1.5 \leq \alpha < -1.0$  (USS),  $-1.0 \leq \alpha < -0.5$  (SSS), and  $-0.5 \leq \alpha \leq 0.0$  (FSS) respectively. Table 1 lists the fractional distribution of sources in detail. Figure 3 confirms the result mentioned above, i.e. the statistical study is complete up to 99% if the adopted flux limitation of the sample corresponds to a spectral index of  $\alpha = -1.5$ . A flux density of 23 mJy at 327 MHz corresponds for  $\alpha_{327}^{1400} = -1.5$  to the NVSS limit of  $S_{1400}$  (2.5 mJy). We take this value as the “flux limitation of the statistical study” (FLSS), which means that less than 2% of the steep-spectrum sources recorded by WENSS are missing in the NVSS. However, all flat-spectrum sources recorded by WENSS being stronger than its sensitivity limit of 18 mJy cannot be missed by the NVSS. Although about 23 000 sources fainter than 23 mJy were cataloged in the WENSS, with most of them having NVSS counterparts, we only use sources stronger than 23 mJy in this study to ensure that the statistical results are based on a unbiased sample.

The resolutions of WENSS and NVSS are  $54'' \times 54''$  cosec $\delta$  and  $45'' \times 45''$  respectively. They are quite similar in size. This means that source intensities can be directly used to calculate spectral indices for most sources.

### 3. Results

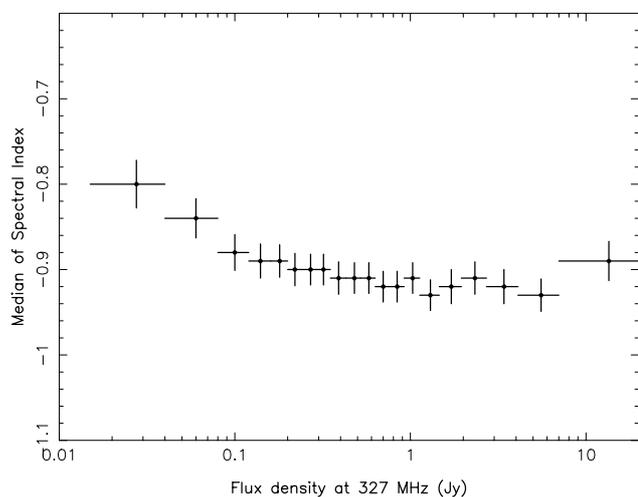
In this section the main results from the cross identification between the WENSS and NVSS are given. The criteria and procedure employed for the source identification are: 1) to search for all NVSS sources within a diameter of  $50''$  centered at every WENSS source; 2) to choose the nearest NVSS source to the central WENSS source; 3) to create a first identification table; 4) to do an inverse search, i.e. within a diameter of  $50''$  centered at each NVSS source identify the nearest WENSS source; 5) to create a second identification table and compare the two tables in order to exclude source pairs with complex corresponding relation. With these criteria and procedure the final cross-identification table used in this study was obtained and spectral indices were calculated accordingly.

Table 2 shows the statistical results on the relationship of flux density and median spectral index, which are also displayed in Fig. 4. Integrated flux densities were used for the spectral index calculations. In Table 2 the first row gives the median flux densities and the boundary flux densities in Jy of each bin at 327 MHz. In the second row of the table the two numbers of each table element represent the numbers of sources found in both catalogs and the medians of the spectral indices respectively. The errors of  $\alpha_{\text{med}}$  in the third line were determined as described in Yule & Kendall (1950). Figure 4 displays this result.

- There is a definite increase of the medians of spectral indices from  $\alpha = -0.89$  to  $\alpha = -0.82$  for median flux densities between 140 mJy to 23 mJy at 327 MHz. This statistical result is based on about 146 000 sources within this low flux density range.
- The spectral indices are nearly constant in the flux density range above 100 mJy up to 7 Jy within the errors. This is contrary to some previously published results as mentioned

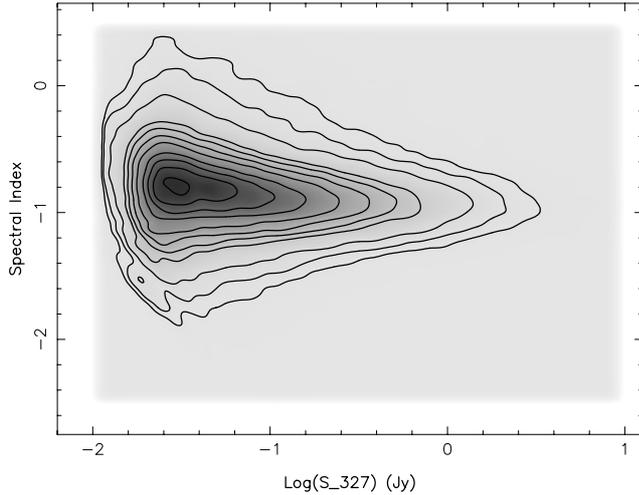
**Table 2.** Statistical results on the relation of spectral index and flux density using WENSS and NVSS samples for sources selected at 327 MHz.

Bin(Jy)	0.03 $_{.040}^{.023}$	0.06 $_{0.08}^{0.04}$	0.10 $_{0.12}^{0.08}$
No./ $\alpha_{\text{med}}$	56261/−0.82	53545/−0.85	23391/−0.88
Error	.033	.029	.028
Bin(Jy)	0.14 $_{0.16}^{0.12}$	0.18 $_{0.20}^{0.16}$	0.22 $_{0.24}^{0.20}$
No./ $\alpha_{\text{med}}$	13106/−0.89	8307/−0.89	5669/−0.90
Error	.027	.027	.025
Bin(Jy)	0.27 $_{0.29}^{0.24}$	0.32 $_{-0.35}^{0.29}$	0.39 $_{0.43}^{0.35}$
No./ $\alpha_{\text{med}}$	4871/−0.90	4103/−0.90	3769/−0.91
Error	.025	.025	.025
Bin(Jy)	0.48 $_{0.52}^{0.43}$	0.58 $_{0.63}^{0.52}$	0.70 $_{0.76}^{0.63}$
No./ $\alpha_{\text{med}}$	2713/−0.91	2404/−0.91	1817/−0.92
Error	.025	.027	.026
Bin(Jy)	0.84 $_{0.92}^{0.76}$	1.03 $_{1.13}^{0.92}$	1.30 $_{1.46}^{1.13}$
No./ $\alpha_{\text{med}}$	1457/−0.92	1169/−0.91	1086/−0.93
Error	.025	.025	.025
Bin(Jy)	1.71 $_{1.95}^{1.46}$	2.33 $_{2.71}^{1.95}$	3.41 $_{4.10}^{2.71}$
No./ $\alpha_{\text{med}}$	818/−0.92	598/−0.91	382/−0.92
Error	.029	.031	.032
Bin(Jy)	5.55 $_{7.0}^{4.1}$	13.5 $_{20.}^{7.0}$	
No./ $\alpha_{\text{med}}$	214/−0.93	109/−0.89	
Error	.028	.035	



**Fig. 4.** Statistical results of the flux density–spectral index relation using the WENSS–NVSS catalogs.

- in the introduction, and partly confirms the result of Kapahi & Kulkarni (1986).
- Below 23 mJy at 327 MHz the incompleteness in cross-identifications limits an accurate statistical study. Above 23 mJy about 98% sources selected at 327 MHz were identified at 1400 MHz. Therefore our results are  $\sim 98\%$  complete and reliable for WENSS sources stronger than 23 mJy.



**Fig. 5.** The contour/grey scale map of the  $\alpha_{\text{med}}-S$  relation of WENSS–NVSS for all the 185 789 source pairs. The map is a  $512 \times 512$  matrix with pixel spacings of  $\Delta S : (10\,000 - 10)/511$  mJy and  $\Delta\alpha : (0.5 - (-2.5))/511$ . The maximum value is 549. Contours start at 9% of the maximum with an interval of 9%.

- At 327 MHz there are 109 sources stronger than 7 Jy (some very complex extended sources were excluded). Although the number is relatively small,  $\alpha_{\text{med}}$  of this bin indicates spectral flattening. The difference between the present result and Kapahi’s result will be discussed in Sect. 4.

Figure 5 shows the contour/grey scale map of the  $\alpha_{\text{med}}-S$  relation of WENSS–NVSS sources for all pairs illustrating the result obtained. The image clearly displays the smooth variation of spectral indices as a function of flux density.

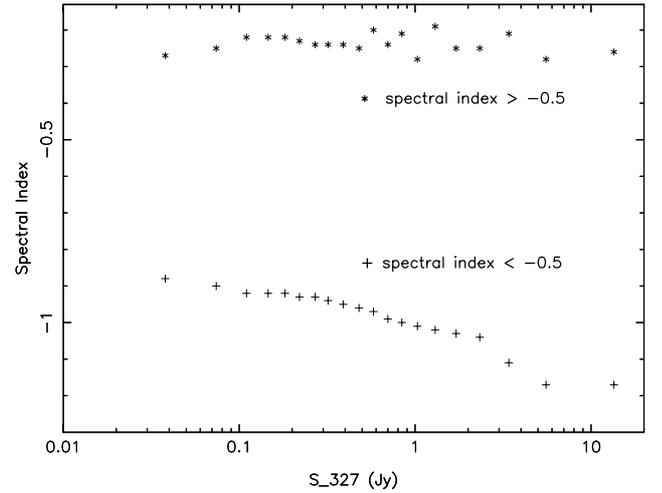
## 4. Discussion

### 4.1. Analysis of the $\alpha_{\text{med}}-S$ variations

Figure 3 shows that with decreasing flux density towards  $\sim 23$  mJy the three fractions vary from 35% to 22% for USS, 57% to 68% for SSS, and 4% to 11% for FSS, respectively. These variations of the fractions for different classes of sources, e.g. the FSS fraction increases and the USS fraction decreases, are partly responsible for the variation of spectral flattening with decreasing flux density in the study of  $\alpha_{\text{med}}-S$  relation. Another contribution to the spectral flattening comes from the slight increase of  $\alpha_{\text{med}}$  of steep-spectrum sources ( $\alpha < -0.5$ ) with decreasing flux density (Fig. 6).

### 4.2. Selection effects and incompleteness

To settle a statistical relationship of flux density and median spectral index, the completeness is very important. There are two kinds of completeness. One is the completeness set by the noise limit of each individual catalog, which for example is 18 mJy for the WENSS and 2.5 mJy for the NVSS. This completeness is important when using each survey individually. Another completeness is related to the statistics of the cross-identification, which reflects how well both catalogs



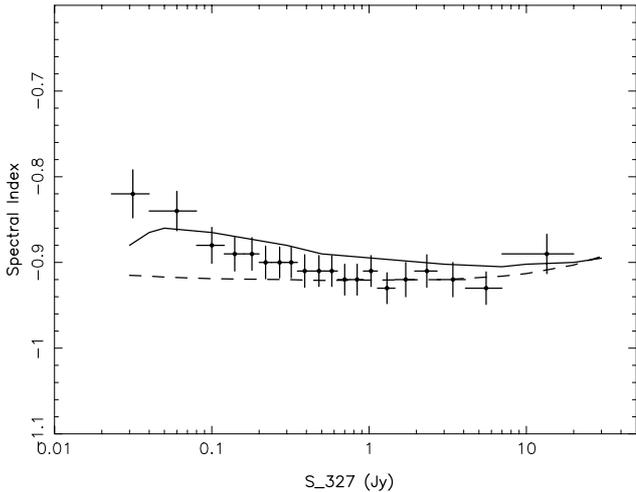
**Fig. 6.** The  $\alpha_{\text{med}}-S$  relations of steep-spectrum and flat-spectrum sources (WENSS–NVSS).

match each other. In general, flat-spectrum sources recorded by the low frequency survey could not be missed by the high frequency survey, whereas some faint steep-spectrum sources near the sensitivity limit of the low frequency survey may be lost by the high frequency survey. This statistical incompleteness, or selection effects, is a more serious limitation for frequency-pairs which have either a relatively large frequency span or the sensitivities of the two surveys do not match each other well. The frequency difference between WENSS and NVSS is not too large and the sensitivities of them are high that they match each other quite well. Our statistical spectral index results using WENSS–NVSS data are complete and reliable for flux densities larger than 23 mJy at 327 MHz (corresponding to  $\alpha_{327}^{1400} = -1.5$ ). A discussion of selection effects with cross-identifications using different catalogs will be given elsewhere (Zhang et al., in press).

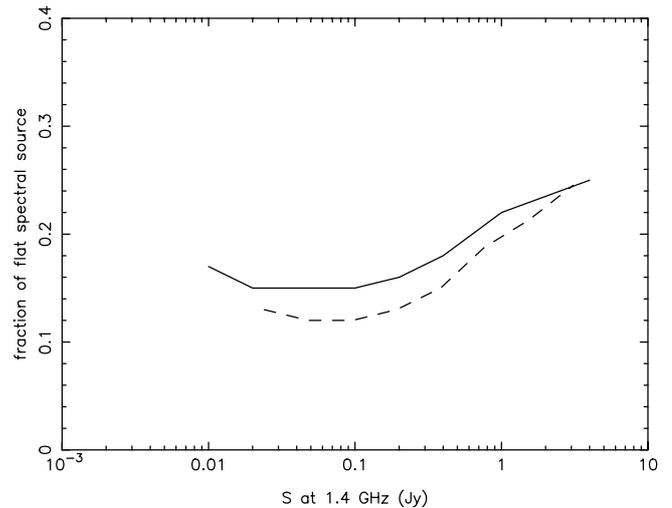
### 4.3. Comparison with radio luminosity models

It is interesting to compare the present results of the  $\alpha_{\text{med}}-S$  relation with predictions made by multi-frequency models of the evolution of the radio luminosity function of extragalactic sources. Kulkarni & Mantovani (1985) used the models of Peacock & Gull (1981, henceforth referred to as PG) for a comparison. These models use separate luminosity and redshift functions for steep- and flat-spectrum sources. Four models are considered: model 1 without and model 2 with a cut-off for the radio luminosity function at  $z = 5$ ,  $q_0 = 0.5$  and models 3 and 4 as models 1 and 2, but with  $q_0 = 0$ .

Kulkarni & Mantovani (1985) found that only a small range of the  $\alpha_{\text{med}}-S$  relation around  $S_{408} \sim 1$  Jy fits the PG models 1 and 4. It is also clear from Kapahi & Kulkarni’s result (1986) that the PG models and the Condon model (1984) fit their result only in a limited range of flux density. The situation is unclear for sources  $S_{408} < 0.1$  Jy, because their samples of available sources is too small. For sources  $S_{408} > 10$  Jy a large deviation from the PG models was noted in Kulkarni & Mantovani’s (1985) comparison. Kapahi & Kulkarni (1986) came to almost



**Fig. 7.** Predicted  $\alpha_{\text{med}}-S$  relations from the model 1 proposed by Peacock & Gull (1981) (full line) and the Condon’s model (dash line) with the new statistical results from this paper (marked by +).



**Fig. 8.** The predicted percentage of flat-spectrum sources at 1.4 GHz (Jackson & Wall 1999) (solid line) and the statistical result (dashed line) using the WENSS and NVSS catalogs.

the same conclusion for sources  $S_{408} > 10$  Jy using the same source sample. The  $S_{408} > 10$  Jy sample they adopted was based on sources from the 408 MHz all-sky survey (Haslam et al. 1982). It seems problematic to calculate spectral indices with 5 GHz data ( $HPBW \sim 3.5'$ ), when considering the effect of the large all-sky survey beam of  $0.85'$ . The beam may contain several sources seen in projection probably with different spectra. Even for the WENSS–NVSS sample with much smaller and almost the same beam sizes, there are cases where one WENSS source corresponds to two NVSS sources. We excluded these sources in the statistics. It seems rather difficult to set up a source sample from surveys with significant differences in resolution.

Figure 7 gives the comparison between our results with the PG model 1 and Condon’s model predictions. A good agreement between the present  $\alpha_{\text{med}}-S$  relation and the PG model 1 is found for  $S_{327}$  sources stronger than 50 mJy. Our results are also in agreement with Condon’s model for sources  $S_{327} > 0.2$  Jy. The fitted range of flux density is much wider than that from previously published results. PG model 1 predicts a significant spectral steepening with decreasing flux density from  $S_{408} < 0.05$  Jy to 0.01 Jy (Kapahi & Kulkarni 1986). Although our result strongly supports the PG model 1 for sources in a wide flux density range, there is no evidence for a spectral steepening of very faint sources. Instead, we found a continuous spectral flattening for faint sources. The situation for sources  $S_{327} < 23$  mJy remains unsettled because of the completeness limit of the samples.

For sources  $S_{327} < 0.1$  Jy, the  $\alpha_{\text{med}}-S$  relation for steep-spectrum sources ( $\alpha < -0.5$ ) is displayed in Fig. 6. The mean spectral index of steep-spectrum sources (USS and SSS) increases (spectral flattening). According to Condon’s source evolution model (1984) this behaviour suggests that  $\alpha_{\text{med}}$  is correlated with luminosity rather than with redshift. However, more detailed studies of faint steep-spectrum sources are needed.

A more recent investigation of extragalactic radio-source evolution published by Jackson & Wall (1999) predicts a population mix at 1.4 GHz according to their dual-population unification scheme. Figure 8 gives the prediction of the percentage of flat-spectrum sources (PFSS, solid line) obtained from their paper together with the new statistical results from the WENSS and NVSS catalogs. This new result is based on a source selection at 1.4 GHz of the frequency pair of WENSS–NVSS, whereas the source selection for Fig. 3 and Table 1 is for 327 MHz. Figure 8 confirms the prediction by Jackson & Wall (1999) in the sense of a varying PFSS with flux density. There is a nearly constant difference ( $\sim 3\%$ ) between their prediction and the observed results. Table 3 lists source numbers recorded by NVSS and WENSS in each bin respectively, the ratio of total identification, and the PFSS ( $\alpha_{327}^{1400} \geq -0.5$ ). Table 3 shows that the completeness of the cross-identification between WENSS and NVSS with source selection at the high frequency end is about 90% on average. The “missing” sources usually have quite a complex structure. Their spectral indices could not be determined with the same accuracy as for compact sources therefore they were excluded from the statistics. This restriction may contribute to the difference between prediction and statistics as shown in Fig. 8.

Comparing Fig. 8 with Fig. 3, two differences should be mentioned. One is that the PFSS from source selection at the low frequency (Fig. 3) is obviously smaller when compared to the PFSS from source selection at the high frequency. This agrees with the result by Jackson & Wall, i.e. the FSS contribution to source counts is smaller at low frequencies ( $< 1$  GHz) than at higher frequencies ( $> 1$  GHz). Another difference is that the PFSS is decreasing or constant with increasing flux density if the source selection is made at the low frequency (Fig. 3), whereas PFSS is increasing with increasing flux density for a source selection at high frequency (Fig. 8). This effect was previously mentioned by Steppe & Gopal-Krishna (1984) and Jackson & Wall (1999).

**Table 3.** Cross-identification of the WENSS and NVSS catalogs with source selection at 1.4 GHz.

Med.flux(1400) (Jy)	NVSS	WENSS	total ident. ratio %	PFSS %
0.024 <sup>0.032</sup> <sub>0.016</sub>	61 378	55 277	90	13
0.048 <sup>0.032</sup> <sub>0.064</sub>	34 492	33 133	96	12
0.090 <sup>0.064</sup> <sub>0.128</sub>	17 954	17 448	97	12
0.192 <sup>0.128</sup> <sub>0.256</sub>	8669	8406	97	13
0.384 <sup>0.256</sup> <sub>0.512</sub>	3511	3371	96	15
0.768 <sup>0.512</sup> <sub>1.024</sub>	1386	1285	93	18.8
1.536 <sup>1.024</sup> <sub>2.048</sub>	417	363	87	21.3
3.072 <sup>2.048</sup> <sub>4.096</sub>	151	125	83	24.5

## 5. Conclusions

The main conclusions from this comparison of the WENSS and the NVSS source catalogs are summarized as follows:

- The median spectral indices increase from  $\alpha_{\text{med}} \sim -0.9$  to  $\alpha_{\text{med}} \sim -0.8$  for  $S_{327}$  flux densities decreasing from 0.1 Jy down to 23 mJy, but are nearly constant in the flux density range above 0.1 Jy up to several Jy. The median spectral indices slightly increase again for  $S_{327}$  above several Jy. This new result is in agreement with published models of the radio luminosity function (RLF) for sources  $S_{327} > 0.1$  Jy and gives constraints to models for sources in the range  $0.023 \text{ Jy} < S_{327} < 0.1 \text{ Jy}$  respectively. This is the first source study down to about 20 mJy at 327 MHz.
- A variation of the fractions of USS, SSS and FSS is partly responsible for the spectral flattening. Another contribution to the spectral flattening comes from the variation of  $\alpha_{\text{med}}$  of steep-spectrum sources ( $\alpha < -0.5$ ) themselves which increases with decreasing flux densities.
- The new results support RLF models by Peacock & Gull (1981) and Condon (1984) for a wide flux density range of sources. Also the PFSS prediction of Jackson & Wall’s model is supported in general.
- The concept of FLSS is adopted for the first time in this study, and the results obtained confirm that this method is useful to ensure statistical completeness.

*Acknowledgements.* X. Zhang thanks for the support from NAO, CAS and the hospitality from MPIfR during his visits in Bonn. This research is also supported by the NSFC.

## References

- Benn, C. R., Grueff, G., Vigotti, M., & Wall, J. V. 1982, MNRAS, 200, 747
- Benn, C. R., Grueff, G., Vigotti, M., & Wall, J. V. 1988, MNRAS, 230, 1
- Condon, J. J. 1984, ApJ, 287, 461
- Condon, J. J., Cotton, W. D., Greisen, E. W., et al. 1998, AJ, 115, 1963 (NVSS)
- Davies, I. M., Little, A. G., & Mills, B. Y. 1973, Aust. J. Phys. Suppl., 28, 1
- Dunlop, J. S., & Peacock, J. A. 1990, MNRAS, 247, 19
- Ficarra, A., Grueff, G., & Tomassetti, G. 1985, A&AS, 59, 255
- Grueff, G., & Vigotti, M. 1979, A&A, 35, 371
- Haslam, C. G. T., Stoffel, H., Salter, C. J., & Wilson, W. E. 1982, A&AS, 47, 1
- Jackson, C. A., & Wall, J. V. 1999, MNRAS, 304, 160
- Kapahi, V. K., & Kulkarni, V. K. 1986, A&A, 165, 39
- Kulkarni, V. K., & Mantovani, F. 1985a, A&AS, 61, 1
- Kulkarni, V. K., & Mantovani, F. 1985b, A&A, 153, 13
- Kulkarni, V. K., Mantovani, F., & Pauliny-Toth, I. I. K. 1990, A&AS, 82, 41
- Large, M. I., Mills, B. Y., Little, A. G., Crawford, D. F., & Sutton, J. M. 1981, MNRAS, 194, 693
- Peacock, J. A., & Gull, S. F. 1981, MNRAS, 196, 611
- Rengelink, R. B., Tang, Y., Bruyn, A. G., et al. 1997, A&AS, 124, 259 (WENSS)
- Robertson, J. G. 1973, Austral. J. Phys., 26, 403
- Steppe, H., & Gopal-Krishna 1984, A&A, 135, 39
- Subrahmanya, C. R., & Kapahi, V. K. 1983, in Early Evolution of the Universe and its Present Structure, ed. G. O. Abell, & G. Chincarini (Dordrecht: Reidel), IAU Symp., 104, 47
- Vigotti, M., Grueff, G., Perley, R., Clark, B. G., & Bridle, A. H. 1989, AJ, 98, 419
- Wall, J. V., Pearson, T. J., & Longair, M. S. 1980, MNRAS, 193, 683
- Wall, J. V., Pearson, T. J., & Longair, M. S. 1981, MNRAS, 196, 597
- Yule, U. G., & Kendall, M. G. 1950, in An Introduction to the Theory of Statistics (London: Charles Griffin & Co. Ltd), 425
- Zhang, X. Z., Reich, W., Reich, P., & Wielebinski, R. 2003, ChJAA, in press