

# The photometric behaviour of SU Dra, 1955–2000

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Received 17 September 2001 / Accepted 07 January 2002

**Abstract.** A multicolour photometric data basis is compiled for SU Dra from 356 Johnson *UBV* and 228 *UBV(RI)<sub>C</sub>* observations reported in this paper, in addition to 483 *BV*, 289 *UBV*, and 85 *UBVRI* observations found in the literature. Mean period, period change  $(4.40 \pm .14) \times 10^{-10}$  day/day, and phase noise are derived from the *V* light curves by a variational procedure minimizing the length of the folded light curve (string length minimization, SLM). From variation of string length in the neighbourhood of minimum, observational error of photometry, observed amplitude, number of observations formulae are derived for the error of period and phase noise by statistical and analytical considerations. Secondary periodic variations are excluded to a few mmag level, indication of amplitude variation (Blazhko-effect) has not been found, however, systematic segregation of light curve segments is reported from different epochs in the descending branch before the brightness rise. Mean light and colour curves are derived from photometry and they are compared with those from integration of spectrophotometric observations. The period change is interpreted in terms of mixing events in the stellar interior.

**Key words.** stars: individual: SU Dra – stars: variables: RR Lyr – stars: fundamental parameters – stars: atmospheres

## 1. Introduction

Theory of the atmosphere of pulsating stars is compelling twofold. A study of the hydrodynamics coupled with radiative transfer in the outermost layers of an RR Lyrae or Cepheid star is astrophysically interesting itself. Since these stars serve as standard candles for distance scales it is important from practical point of view as well to have an adequate theory of their atmospheres e.g. for determining their absolute brightness. The aim of the present paper is to report on Johnson *UBV* and *UBV(RI)<sub>C</sub>* observations of the RR<sub>ab</sub> variable SU Dra and to determine accurate light curves which are challenging to theoretical interpretation.

In the observations as long segments of the light curve were observed as possible and a full phase coverage was set as goal in order to be able to examine congruency of the consecutive cycles, eventual variation of the light curve, period change and phase noise. The light curves are analysed from the observations in addition to the available photoelectric and spectrophotometric light curves found in the literature. The answer will be sought for the question to what extent is the *V* light curve a monophasic and coherent signal. A variational procedure is used to obtain the period with its change and phase shifts of the light

curve segments by minimizing the length of the folded light curve. Since one of the main interests of this paper lies in analysis and homogenisation of large body of observations the transformation of instrumental magnitudes to *UBV(RI)<sub>C</sub>* is described briefly.

Section 2 describes the observations by which the *UBV* or *UBV(RI)<sub>C</sub>* data of number 584 were obtained and the source of *UBV* or *UBVRI* photometry of SU Dra found in the literature: discussion and homogenisation of a material consisting 1441 two, three, or five colour observations of different observers. The procedure of period search, error analysis, and the mean *UBV(RI)<sub>C</sub>* light curve will be presented in Sect. 3. New period and period change rate will be derived, error estimations are given. In Sects. 4, 5 the results will be discussed and the conclusions will be drawn. The present paper will document the broad-band photometric behaviour of SU Dra for almost half a century and a forthcoming paper is planned to draw theoretical conclusions from the present light curves.

## 2. The observations

With the 1 m Ritchey-Chretien telescope of the Konkoly Observatory at Piszkestető SU Dra and the comparison stars BD +67°708, BD +68°655 were observed from 1982 on. Usually BD +68°655 was observed less frequently

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**Table 1.** Logbook of the present observations.

HJD–2 400 000	$n$	exp. (s)	$\sigma_V$ [mag]	$k$
44989.41–.71 <sup>a</sup>	80	30, 30, 30	.016	25
45051.34–.44 <sup>a</sup>	16	30, 30, 30	.039	26
45052.31–.56 <sup>a</sup>	115	30, 30, 30	.018	27
45406.33–.55 <sup>a</sup>	115	30, 30, 30	.007	28
45407.36–.57 <sup>a</sup>	30	30, 30, 30	.016	29
49817.35–.45 <sup>b</sup>	19	100, 20, 10, 5, 5	.021	42
49918.40–.48 <sup>b</sup>	10	150, 30, 15, 7, 7	.032	43
49919.35–.39 <sup>b</sup>	10	30, 8, 2, 1, 1	.015	44
49919.50–.56 <sup>b</sup>	15	30, 8, 10, 5, 5	.015	44
49921.34–.35 <sup>b</sup>	5	30, 8, 10, 5, 5	.003	45
50939.38–.48 <sup>b</sup>	15	240, 100, 40, 15, 10	—*	46
51024.37–.44 <sup>b</sup>	4	200, 100, 40, 15, 10	.015	47
51678.38–.57 <sup>c</sup>	36	90, 20, 10, 5, 5	.003	48
51680.36–.57 <sup>c</sup>	55	90, 20, 10, 5, 5	.013	49
51681.33–.58 <sup>c</sup>	14	90, 10, 5, 5	.007	50
51682.32–.48 <sup>c</sup>	45	90, 20, 10, 5, 5	.005	51

<sup>a</sup> *UBV* photoelectric photometer.

<sup>b</sup> *UBV(RI)<sub>C</sub>* Wright camera.

<sup>c</sup> *UBV(RI)<sub>C</sub>* Photometrics camera.

\* Only two pair observations of the comparison stars are available.

as check star: it was observed at least every 2 hours when the sky was stable while at less stable sky conditions comp<sub>1</sub>, comp<sub>2</sub>, 2 × 3 SU Dra was the measuring cycle. The two comparison stars were selected on the basis that Spinrad (1961) used BD +67°708 while Oláh & Szeidl (1978) used BD +68°655. They are of different colours and their distances to SU Dra are 43' and 32' respectively. A few attempts were made to tie the comparison stars in the *UBV(RI)<sub>C</sub>* system, two of them were successful. Table 1 gives the logbook of the observations. Column 1 gives the JD interval and the kind of the observations i.e. *UBV* indicates that an uncooled integrating photoelectric photometer was used with tube of type 1P 21 and appropriate Schott filters, *UBV(RI)<sub>C</sub>* indicates the use of a Peltier cooled CCD camera from Wright Instruments or Photometrics (Bakos 2000), Col. 2 contains the number  $n$  of the multicolour observations. Column 3 shows the exposure times in order  $U, B, \dots$ , Col. 4 gives the estimated standard deviation (SD)  $\sigma_V$  for  $V$  band to characterize the photometric quality of the night. The last column is the reference number  $k$  of the light curve segment.

Table 2 is a summary of the broad band multicolour photometric observations found in the literature which could be, at least partly, homogenized with the present observations.

### 2.1. The transformations of instrumental magnitudes *ubvri* to *UBV(RI)<sub>C</sub>*

The photoelectric observations were reduced by standard procedures (Hardie 1962). The formulae

$$C_J = \kappa_j c_j - k_j X + K_j \quad (1)$$

were assumed where  $C$  is a colour index,  $J = B - V$  or  $U - B$ ,  $j = b - v$  or  $u - b$ ,  $c_j$  is the instrumental colour index,

**Table 2.** The broad band multicolour observations from the literature and their reference number  $k$  in the paper.

HJD	$n$	$k$	HJD	$n$	$k$		
2 400 000+			2 400 000+				
36152 <sup>a</sup>	25	<i>BV</i>	1	42415 <sup>a</sup>	48	<i>BV</i>	19
36163 <sup>a</sup>	5	<i>BV</i>	2	42452 <sup>a</sup>	67	<i>UBV</i>	20
36164 <sup>a</sup>	40	<i>BV</i>	3	42454 <sup>a</sup>	69	<i>UBV</i>	21
36187 <sup>a</sup>	22	<i>BV</i>	4	42532 <sup>a</sup>	50	<i>UBV</i>	22
36199 <sup>a</sup>	21	<i>BV</i>	5	42948 <sup>a</sup>	18	<i>BV</i>	23
36203 <sup>a</sup>	8	<i>BV</i>	6	43204 <sup>a</sup>	23	<i>BV</i>	24
36617 <sup>a</sup>	17	<i>BV</i>	7	46846 <sup>c</sup>	7	<i>UBVRI</i>	30
36619 <sup>a</sup>	76	<i>BV</i>	8	46847 <sup>c</sup>	6	<i>UBVRI</i>	31
36624 <sup>a</sup>	11	<i>BV</i>	9	46848 <sup>c</sup>	23	<i>UBVRI</i>	32
36626 <sup>a</sup>	90	<i>BV</i>	10	46849 <sup>c</sup>	6	<i>UBVRI</i>	33
36645 <sup>a</sup>	36	<i>BV</i>	11	47121 <sup>c</sup>	6	<i>UBVRI</i>	34
36983 <sup>b</sup>	1	<i>UBV</i>	12	47122 <sup>c</sup>	5	<i>UBVRI</i>	35
36984 <sup>b</sup>	7	<i>UBV</i>	13	47123 <sup>c</sup>	4	<i>UBVRI</i>	36
36988 <sup>b</sup>	13	<i>UBV</i>	14	47124 <sup>c</sup>	6	<i>UBVRI</i>	37
37036 <sup>b</sup>	30	<i>UBV</i>	15	47198 <sup>c</sup>	8	<i>UBVRI</i>	38
37044 <sup>b</sup>	47	<i>UBV</i>	16	47226 <sup>c</sup>	9	<i>UBVRI</i>	39
37045 <sup>b</sup>	5	<i>UBV</i>	17	47227 <sup>c</sup>	4	<i>UBVRI</i>	40
42403 <sup>a</sup>	43	<i>BV</i>	18	47228 <sup>c</sup>	1	<i>UBVRI</i>	41

<sup>a</sup> Oláh & Szeidl (1978).

<sup>b</sup> Spinrad (1961).

<sup>c</sup> Liu & Janes (1989).

$\kappa_j$  is an instrumental constant,  $k_j$  is the colour extinction coefficient, the second order terms were neglected since their values were very small if it was possible to determine them from the observations (and they did not change significantly the final magnitudes),  $X$  is the air mass,  $K_j$  is the zero point of the magnitude scale. In addition to (1)

$$V = v - k_v X + \epsilon(B - V) + K_v \quad (2)$$

was assumed,  $\epsilon$  is defined in Hardie (1962).

The primary results of CCD observations are *ubvri* values, therefore, formulae of type (2) with eventual colour terms like e.g.

$$U = u - k_u X + \alpha(u - b) + \beta \quad (3)$$

and similar formulae for  $B, R_C, I_C$  are frequently used (e.g. Neely et al. 2000; Phelps 1997; Bragaglia 1997),  $\alpha, \beta$  are different constants for each colour. In reducing the present multicolour observations, however, the conservative formulae (1), (2) were used throughout (supplemented with  $J = V - R_C, R_C - I_C, j = v - r, r - i$ ) as suggested by Brocato (1994), Carraro & Palat (1994). This choice was motivated by the reduction of the observations on JD = 2 449 921, 2 451 681 which gave for the comparison stars  $V, B - V, U - B$  values agreeing within the observational error with those of Spinrad (1961), Oláh & Szeidl (1978), Liu & Janes (1989) & the *Hipparcos* data (ESA, 1997). On the other hand formulae of type (3) gave wrong colour indices in these nights for the comparison star. This result indicates the worse representation of colour extinction and colour index zero point by (3) in spite of the fact that (1)–(3) are based on the same simplified assumption of atmospheric extinction (Hardie 1962).

From the photoelectric observations the instrumental values *ubv* were obtained in the traditional way.

The values *ubvri* of the CCD observations were obtained by standard IRAF<sup>1</sup> reduction procedures: bias, flat field correction, and aperture photometry (package DAOPHOT, procedure PHOT) were applied because the field is not crowded. The dark current was found negligible, thus dark correction was not made. At the high signal to noise ratio the error of the instrumental magnitudes estimated by IRAF was negligible, characteristically  $O(0.001)\text{mag}$ .

The observations of SU Dra were reduced by interpolation using the two comparison stars since both stars lie in the proximity of SU Dra. The formulae

$$C_J^{(\text{SU Dra})}(t) = \kappa_j \Delta c_j(t) + C_J^{(\text{comp. star})} \quad (4)$$

$$V^{(\text{SU Dra})}(t) = \Delta v(t) + \epsilon \Delta C_{B-V}(t) + V^{(\text{comp. star})} \quad (5)$$

were used, if possible to both comparison stars, where  $\Delta c_j(t)$ ,  $\Delta v(t)$ ,  $\Delta C_{B-V}(t)$  are the instrumental colour,  $v$  magnitude and  $B-V$  differences of SU Dra and comparison stars, linear time ( $t$ ) variation was assumed between two measurements of the comparison stars. The final light curves  $U(t)$ ,  $B(t)$ , ... were obtained by trivial additions. The coefficients  $\kappa_j$ ,  $k_j$ ,  $\epsilon$  were determined from the observations of the comparison stars in one night. From the values of  $k_j$  it was obvious that the magnitude differences from air mass differences were negligible ( $<0.001$  mag) because of the proximity of the stars. This way of reduction provided particularly homogeneous magnitude and colour values. The most uncertain factor in the reductions, the zero point of the magnitude scale, was cancelled by (4), (5).

The random scatter of the final light curves originates from the variable sky conditions, it could be estimated from the SDs  $\sigma_V$ ,  $\sigma_{B-V}$ , ... of the two comparison stars which was determined for each night from pair observations at same air mass.  $\sigma_V$  is given in Table 1. The lower and upper limits for the SD of  $B-V$ ,  $U-B$ ,  $V-R_C$ ,  $R_C-I_C$  were  $0.001-0.038$ ,  $0.008-0.126$ ,  $0.008-0.047$ ,  $0.007-0.023$  mag, respectively. The weighted means of  $\sigma_V$ ,  $\sigma_{B-V}$ , ... were  $0.015$ ,  $0.023$ ,  $0.054$ ,  $0.018$ ,  $0.014$  mag, respectively. A further indicator of the quality of the light curves is the averaged magnitude difference of SU Dra determined from BD +67°708 and BD +68°655 which was found in consensus with the previous values. If structures of the light and colour curves are found in excess to these SD limits, especially the repetitive ones, they must be regarded as intrinsic features containing physical information on the pulsation of SU Dra. From the final list those observations were removed which were obviously disturbed by atmospheric transparency changes, cirri, e.g. 20 points between 2 449 919.39 – .49.

## 2.2. The comparison stars

When the sky was uniformly clear by eye inspection and SU Dra was not in ascending branch occasionally

<sup>1</sup> IRAF is distributed by the NOAO operated by the AURA Inc. under contract with the NSF.

$UBV(RI)_C$  secondary standard stars were observed from the Selected Area stars (Landolt 1983; Menzies et al. 1991) to determine the  $UBV(RI)_C$  magnitudes of the comparison stars by using a least square solution for formulae (1), (2). Among these nights JD = 2 451 681 was of good quality and JD = 2 449 921 was acceptable i.e. the SD of the five colour observations of the SA stars did not exceed a reasonable limit (e.g. 0.02 mag in  $V$ ). Table 3 summarizes the magnitudes from the literature and the present work. The standard errors were estimated from 8 five colour observations of BD +67°708 and BD +68°655 respectively, they represent a 68 percent confidence level. The agreement of the magnitudes with those of other sources is satisfactory.

The magnitude differences of the two comparison stars were taken from Table 3 (i.e.  $\Delta V = 0.291$ ,  $\Delta(U-B) = 0.913$ , ... etc.) when for every night of Table 1  $\kappa_j$  and  $\epsilon$  were determined from the values  $\Delta v$ ,  $\Delta(u-b)$ , ... etc.

The *Hipparcos* and present observations (95 pair values of five colours at smaller air mass differences than 0.01) do not indicate a variability of the comparison stars.

## 2.3. The observational data basis

$UBV$  observations of SU Dra were published by Spinrad (1961), Oláh & Szeidl (1978) who used comparison stars and by Fitch et al. (1966) who tied their observations in the Johnson  $UBV$  system in each night.  $UBVRI$  observations were published by Liu & Janes (1989), one of their comparison stars was BD +67°708.

The  $V$  light curve segments of Spinrad (1961), Oláh & Szeidl (1978), Liu & Janes (1989) are congruent with the present observations, they could be incorporated in the analysis while those of Fitch et al. (1966) had to be omitted because a zero point shift  $\approx 0.05$  mag would be necessary to reach an approximate congruency, a rather big random scatter was found compared to the averaged light curve and their segment of JD = 2 438 411 deviates significantly from all other observations. The final list for searching period and phase noise consisted of  $N_V = 1441$   $V$  magnitudes in 51 segments<sup>2</sup>, the ratio of the points on the ascending and descending branch is 2:1. A short description of the  $V$  data set:  $V_{\max} = 9.199$ ,  $V_{\min} = 10.262$ , (magnitude averaged)  $\bar{V} = 9.718$  with a standard deviation .354 and skewness .026.

The  $B$  and  $U$  observations of Oláh & Szeidl (1978) would be approximately congruent with those of the other three sources if zero point shifts  $\approx -0.08$  mag and  $\approx 0.13$  mag were applied. Since their observational goal was to construct O-C curves and the sky quality of the Budapest observations was unfavourable to  $U$  and  $B$  it seemed appropriate to omit this material from the present study which is aimed to compile light and colour curves for determining physical parameters of SU Dra. By this

<sup>2</sup> The  $V$  file is available via anonymous ftp from: <ftp://ftp.konkoly.hu/pub/download/barcza/suVsu.dat>

**Table 3.**  $UBV(RI)_C$  values of comparison stars BD+67°708 and BD+68°655.

$V$	$U - B$	$B - V$	$V - R_C$	$R_C - I_C$	reference
BD+67°708					
$8.978 \pm .019$	$.061 \pm .015$	$.232 \pm .008$			Spinrad (1961)
$8.950 \pm .015$		$.255 \pm .019$			<i>Hipparcos</i> , ESA 1997
$8.964 \pm .002$	$.079 \pm .002$	$.244 \pm .001$	$.150 \pm .002^*$	$.151 \pm .002^*$	Liu & Janes (1989)
$8.963 \pm .028$	$.053 \pm .009$	$.215 \pm .010$	$.131 \pm .006$	$.171 \pm .003$	present work
BD+68°655					
9.26	.98	1.05			Oláh & Szeidl (1978)
$9.230 \pm .015$		$1.135 \pm .035$			<i>Hipparcos</i> , ESA 1997
$9.254 \pm .027$	$.966 \pm .026$	$1.106 \pm .020$	$.647 \pm .024$	$.530 \pm .011$	present work

\* Johnson  $R$  and  $I$  colours.

omission the final  $B$  and  $U$  files contained 762 and 750 data respectively.

Johnson  $R$  and  $I$  observations were published by Liu & Janes (1989). Their values and the present  $R_C$ ,  $I_C$  values were unified in common files since systematic difference was not found. The  $R_C$  and  $I_C$  files consist of 310 and 290 data respectively.

The results of the present observations are given in Tables 4, 5. They give 40–80 percent of the broad band photometry used in the paper.

Spectrophotometry was published by Oke et al. (1962) covering the wave length range from ultraviolet to visual which could be converted to Johnson  $UBV$  colours by using the Vilnius filters of Azusienis & Strayzys (1969) and spectrophotometry to the Vega calibration of Tüg et al. (1977). A phase shift of 0.02 was applied to be in accordance with the phasing in the present paper. The duration of a spectrophotometric integration was some 14 min which has a smoothing effect in the phases when the brightness changes rapidly. The computed  $UBV$  magnitudes were not drawn into the homogenized observational material, however, these light curves served as an independent check for those obtained from  $UBV$  photometry.

### 3. Periods, phase noise, mean light and colour curves

General problems of period finding are beyond the scope of the present paper, to them a source of references is given e.g. by Schwarzenberg-Czerny (1999). This section will summarize the results which were obtained from the most populous data set. It is in  $V$ : 51 segments covering 6.36 days are distributed over 15 530 days (i.e. approximately  $2.35 \times 10^4$  cycles of pulsation). A Fourier analysis gave a forest of peaks around the true period because of the very sparse sampling: practically the bad spectral window (Deeming 1975) determined the output. Therefore, a method had to be applied in which the first step is to fold the light curve. String length minimization (SLM, Burke et al. 1970) and phase dispersion minimization (PDM, Stellingwerf 1978) represent two variants of this technique. In PDM the magnitudes of a folded light curve are binned and an average is calculated to which a scatter is determined. In SLM the length of a light curve is approximated

by a sum of strings. The next step is to minimize the scatter or length respectively as a function of the period. Burke et al. (1970) applied SLM for magnetic variables but they did not give details. Dworetzky (1983) applied and discussed SLM for binary stars. In this case strictly periodic radial velocity curve can be assumed i.e. the model of the data to be analyzed is the projection of an orbital motion which is different from that of a pulsating star where the observed quantity is the brightness in a photometric band. This comes from coupled hydrodynamical, radiative processes, integrals over differentially moving layers and wavelength. Therefore, a strict periodicity of the signal can be assumed as exception rather than rule. The binning and averaging step of PDM can cause minor systematic errors. These steps are not part of SLM, therefore SLM is to the observed quantities as close as possible (Stetson 1996).

#### 3.1. Period search by string length minimization

The method applied here is an adaptation of SLM for the present data set. It will be described in some detail for clarity and highlighting the additions which may rely upon wider interest: handling phase shifts from period change, phase noise, normalization, error estimations, and some useful relations indicating whether the signal monoperoiodic is. Four steps are involved.

- (1) The middle points  $t_i$ ,  $1 \leq i \leq N_V$  of a magnitude observation  $V$  were projected on the phase axis  $\phi$  by the saw-tooth function of period  $P^{(0)}$  i.e.

$$\phi = (t_i - \delta_k - t_0)/P^{(0)} - [(t_i - \delta_k - t_0)/P^{(0)}]_{\text{ent}} \quad (6)$$

where  $[...]_{\text{ent}}$  denotes the integer part function,  $t_0 = \text{HJD}_0$  is the starting epoch, by its suitable choice  $\phi = 0$  must be reached at one of the maxima,  $|\delta_k/P^{(0)}| \ll .5$  is the phase shift of the  $k$ th light curve segment,  $\delta = (\delta_1, \dots)$ ,  $\delta_k$  is the well known O–C value of the light curve segment  $k$  for the constant period  $P^{(0)}$  if  $t_k$  is the epoch of a maximum brightness. Period and  $\delta_k$  will be given in units of days.

- (2) The magnitudes  $V(\phi_i) = V(t_i)$  were reordered according to increasing  $\phi_j$  ( $\phi_j \geq \phi_{j-1}$  for any  $2 \leq j \leq N_V$ )

**Table 4.** *UBV* magnitudes of SU Dra from the present observations: light curve segments  $k = 25–29$ . The fraction of HJD is given for *V*, by adding .0004 and .0008 to it the epochs of *B* and *U* are obtained.

HJD	<i>V</i>	<i>B</i>	<i>U</i>												
2 444 989+				.3684	9.754	10.050	10.059	.5078	9.224	9.333	9.426	.4525	9.537	9.724	9.723
.4075	9.941	10.292	10.295	.3892	9.790	10.161	10.172	.5090	9.228	9.321	9.421	.4537	9.528	9.700	9.720
.4086	9.956	10.296	10.310	.3906	9.832	10.144	10.188	.5102	9.215	9.329	9.430	.4548	9.513	9.694	9.704
.4098	9.951	10.306	10.299	.3919	9.798	10.132	10.152	.5160	9.244	9.355	9.446	.4559	9.502	9.674	9.696
.4110	9.952	10.310	10.319	.4341	9.865	10.213	10.172	.5172	9.229	9.352	9.455	.4619	9.425	9.557	9.598
.4122	9.950	10.307	10.313	.4355	9.859	10.184	10.179	.5184	9.231	9.358	9.447	.4630	9.404	9.548	9.596
.4134	9.965	10.312	10.328	.4369	9.861	10.170	10.177	.5196	9.239	9.360	9.454	.4642	9.382	9.531	9.577
.4145	9.961	10.323	10.336	2 445 052+				.5208	9.247	9.372	9.472	.4653	9.373	9.516	9.569
.4158	9.973	10.317	10.345	.3056	10.065	10.423	10.432	.5220	9.252	9.371	9.479	.4665	9.358	9.500	9.546
.4170	9.961	10.325	10.328	.3067	10.065	10.421	10.420	.5232	9.256	9.378	9.473	.4676	9.341	9.473	9.530
.4181	9.963	10.328	10.342	.3080	10.067	10.437	10.422	.5244	9.253	9.380	9.476	.4688	9.327	9.450	9.522
.4194	9.968	10.329	10.343	.3092	10.063	10.436	10.423	.5256	9.268	9.401	9.497	.4700	9.320	9.440	9.525
.4206	9.969	10.334	10.345	.3103	10.067	10.445	10.390	.5268	9.273	9.405	9.498	.4711	9.311	9.425	9.494
.4218	9.977	10.329	10.353	.3231	10.083	10.435	10.435	.5327	9.312	9.429	9.541	.4723	9.291	9.414	9.488
.4241	9.979	10.341	10.364	.3243	10.084	10.431	10.453	.5339	9.316	9.436	9.552	.4777	9.255	9.355	9.443
.4461	9.999	10.387	10.361	.3255	10.086	10.433	10.428	.5351	9.315	9.449	9.568	.4789	9.241	9.355	9.431
.4511	10.007	10.384	10.368	.3267	10.091	10.432	10.428	.5363	9.316	9.451	9.546	.4800	9.243	9.353	9.431
.4524	10.015	10.383	10.363	.3278	10.088	10.440	10.435	.5375	9.318	9.455	9.556	.4812	9.242	9.355	9.426
.4535	10.020	10.393	10.359	.3358	10.116	10.480	10.430	.5387	9.321	9.456	9.549	.4823	9.240	9.354	9.433
.4721	10.047	10.476	10.406	.3370	10.145	10.494	10.480	.5399	9.323	9.456	9.564	.4835	9.235	9.351	9.443
.4733	10.050	10.454	10.430	.3382	10.124	10.492	10.492	.5411	9.327	9.461	9.559	.4847	9.236	9.349	9.430
.4745	10.052	10.463	10.400	.3397	10.129	10.492	10.488	.5422	9.325	9.465	9.576	.4858	9.236	9.349	9.440
.4758	10.051	10.466	10.421	.3406	10.116	10.496	10.494	.5434	9.338	9.419	9.529	.4870	9.225	9.350	9.450
.4769	10.062	10.439	10.403	.3486	10.196	10.557	10.574	.5515	9.370	9.504	9.606	.4881	9.238	9.357	9.437
.4780	10.067	10.432	10.403	.3497	10.160	10.575	10.555	.5527	9.360	9.506	9.594	.4941	9.230	9.342	9.436
.4793	10.064	10.428	10.401	.3509	10.161	10.551	10.567	.5538	9.358	9.513	9.625	.4953	9.232	9.339	9.429
.4805	10.071	10.436	10.423	.3521	10.160	10.537	10.553	.5550	9.368	9.519	9.603	.4964	9.229	9.343	9.425
.4817	10.065	10.443	10.394	.3533	10.166	10.561	10.570	.5562	9.369	9.526	9.635	.4976	9.208	9.316	9.411
.4829	10.069	10.433	10.407	.3611	10.160	10.534	10.512	.5574	9.369	9.529	9.624	.4987	9.242	9.351	9.445
.4841	10.075	10.436	10.408	.3623	10.165	10.569	10.576	.5586	9.381	9.531	9.633	.4999	9.244	9.355	9.455
.4853	10.069	10.440	10.410	.3635	10.179	10.565	10.573	.5598	9.385	9.543	9.644	.5011	9.249	9.361	9.456
.4864	10.072	10.442	10.410	.3647	10.178	10.576	10.554	.5610	9.394	9.552	9.641	.5023	9.253	9.361	9.471
.4877	10.074	10.441	10.418	.3659	10.173	10.573	10.571	.5622	9.390	9.563	9.662	.5034	9.259	9.370	9.460
.4888	10.072	10.441	10.401	.3789	10.219	10.610	10.609	2 445 406+				.5045	9.269	9.387	9.490
.5242	10.074	10.436	10.403	.3801	10.221	10.613	10.613	.3289	10.131	10.509	10.505	.5125	9.282	9.401	9.501
.5255	10.066	10.438	10.404	.3813	10.222	10.614	10.627	.3298	10.127	10.502	10.506	.5137	9.290	9.413	9.516
.5267	10.066	10.435	10.397	.3825	10.214	10.608	10.623	.3310	10.135	10.509	10.505	.5148	9.299	9.421	9.518
.5278	10.067	10.428	10.404	.3837	10.222	10.607	10.606	.3322	10.143	10.520	10.502	.5160	9.305	9.431	9.530
.5291	10.060	10.430	10.397	.3944	10.246	10.624	10.627	.3334	10.146	10.528	10.521	.5172	9.308	9.439	9.532
.5303	10.057	10.432	10.410	.3966	10.248	10.626	10.616	.3434	10.174	10.556	10.542	.5183	9.305	9.444	9.546
.5315	10.063	10.427	10.398	.3968	10.233	10.612	10.595	.3446	10.172	10.543	10.530	.5195	9.323	9.463	9.573
.5327	10.059	10.438	10.405	.3980	10.228	10.591	10.575	.3457	10.173	10.559	10.525	.5206	9.328	9.478	9.540
.5338	10.062	10.429	10.403	.3992	10.217	10.599	10.583	.3469	10.181	10.558	10.560	.5218	9.310	9.442	9.534
.5350	10.058	10.432	10.403	.4004	10.214	10.599	10.581	.3480	10.174	10.561	10.555	.5229	9.319	9.448	9.549
.5361	10.060	10.427	10.409	.4016	10.222	10.600	10.599	.3587	10.200	10.587	10.558	.5311	9.338	9.468	9.564
.5374	10.052	10.426	10.404	.4028	10.216	10.593	10.567	.3595	10.204	10.587	10.551	.5322	9.356	9.487	9.570
.5386	10.053	10.427	10.404	.4039	10.212	10.588	10.580	.3607	10.212	10.573	10.548	.5334	9.354	9.499	9.585
.5397	10.049	10.427	10.392	.4051	10.215	10.600	10.589	.3622	10.201	10.580	10.570	.5345	9.367	9.508	9.581
.5410	10.065	10.424	10.392	.4387	10.022	10.329	10.272	.3630	10.207	10.589	10.568	.5357	9.376	9.514	9.597
.5672	10.080	10.435	10.422	.4399	10.002	10.314	10.281	.3723	10.212	10.598	10.596	.5369	9.384	9.537	9.613
.5688	10.084	10.443	10.423	.4411	9.975	10.288	10.253	.3735	10.210	10.602	10.601	.5380	9.399	9.545	9.618
.5699	10.091	10.442	10.455	.4423	9.959	10.271	10.224	.3750	10.225	10.606	10.607	.5392	9.395	9.541	9.619
.5712	10.090	10.445	10.435	.4434	9.935	10.243	10.189	.3758	10.213	10.606	10.625	.5403	9.399	9.563	9.629
.5724	10.099	10.445	10.448	.4492	9.822	10.087	9.999	.3770	10.227	10.609	10.620	.5415	9.403	9.576	9.654
.5895	10.125	10.477	10.476	.4504	9.783	10.041	9.937	.3851	10.222	10.608	10.582	.5499	9.414	9.579	9.658
.5908	10.125	10.484	10.476	.4516	9.748	9.994	9.871	.3863	10.230	10.615	10.586	.5511	9.421	9.581	9.629
.5920	10.132	10.487	10.473	.4528	9.714	9.965	9.845	.3875	10.222	10.615	10.559	.5523	9.418	9.580	9.667
.5932	10.138	10.486	10.471	.4540	9.679	9.929	9.817	.3886	10.221	10.610	10.574	.5538	9.425	9.610	9.673
.5943	10.136	10.485	10.509	.4552	9.654	9.895	9.794	.3891	10.220	10.602	10.575	.5545	9.439	9.606	9.676
.6192	10.169	10.541	10.528	.4564	9.633	9.865	9.784	.3976	10.216	10.589	10.581	2 445 407+			
.6203	10.172	10.553	10.520	.4575	9.618	9.844	9.770	.3992	10.215	10.587	10.579	.3616	9.826	10.145	10.172
.6216	10.167	10.557	10.541	.4587	9.600	9.826	9.772	.4003	10.221	10.585	10.583	.3627	9.823	10.145	10.170
.6228	10.166	10.559	10.546	.4599	9.590	9.809	9.765	.4012	10.214	10.584	10.585	.3639	9.833	10.153	10.193
.6240	10.182	10.563	10.526	.4659	9.562	9.740	9.736	.4023	10.211	10.580	10.565	.3650	9.832	10.154	10.172
.6402	10.205	10.632	10.625	.4671	9.534	9.724	9.717	.4042	10.211	10.571	10.564	.3662	9.835	10.151	10.183
.6415	10.220	10.600	10.576	.4683	9.521	9.709	9.726	.4054	10.216	10.565	10.546	.3745	9.855	10.182	10.191
.6427	10.221	10.611	10.575	.4695	9.513	9.685	9.685	.4069	10.210	10.560	10.560	.3757	9.854	10.181	10.178
.6438	10.221	10.602	10.581	.4707	9.493	9.663	9.712	.4081	10.207	10.550	10.548	.3768	9.854	10.176	10.183
.6451	10.222	10.611	10.587	.4719	9.482	9.645	9.666	.4089	10.194	10.511	10.494	.3780	9.851	10.179	10.170
.6619	10.231	10.607	10.577	.4731	9.463	9.622	9.647	.4161	10.119	10.449	10.382	.3795	9.861	10.182	10.168
.6630	10.232	10.606	10.569	.4743	9.443	9.600	9.637	.4172	10.100	10.439	10.375	.4415	9.931	10.291	10.245
.6643	10.233	10.611	10.474	.4755	9.425	9.577	9.608	.4184	10.097	10.424	10.382	.4427			

**Table 5.**  $UBV(RI)_C$  magnitudes of SU Dra from the present observations: light curve segments  $k = 42 - 51$ . The fraction of HJD is given for  $V$ , the epochs of  $B$ ,  $U$ ,  $R_C$ ,  $I_C$ , are obtained by adding the values after 24...+ to it.

HJD	$V$	$B$	$U$	$R_C$	$I_C$	HJD	$V$	$B$	$U$	$R_C$	$I_C$	HJD	$V$	$B$	$U$	$R_C$	$I_C$
2 449 817+,	-.0006,	-.0022,	.0005,	.0009		2 451 024+,	-.0020,	-.0058,	.0009,	.0015		.5034	9.239	9.342	9.388	9.139	8.974
.3486	10.210	10.635	—	9.915	9.582	.3709	9.969	10.388	10.329	—	—	.5108	9.246	9.345	9.408	9.145	8.981
.3252	10.214	10.638	10.573	9.933	9.577	.3969	10.004	10.407	10.374	9.712	9.350	.5129	9.258	9.352	9.401	9.143	8.986
.3562	10.205	10.617	10.632	9.905	9.565	.4201	10.053	10.434	10.409	9.739	9.374	.5173	9.261	9.369	9.432	9.163	8.986
.3722	10.114	10.499	10.507	9.802	9.499	.4427	10.061	10.480	10.439	9.731	9.390	.5195	9.268	9.370	9.427	9.152	8.990
.3765	10.055	10.458	10.419	9.791	9.500	2 451 678+,	-.0003,	-.0011,	.0002,	.0004		.5263	9.288	9.403	9.470	9.188	9.003
.3804	10.000	10.412	10.373	9.745	9.462	.3814	10.169	10.555	10.454	9.857	9.521	.5284	9.291	9.411	9.464	9.197	9.006
.3840	9.964	10.336	10.284	9.706	9.419	.3836	10.171	10.564	10.462	9.851	9.526	.5306	9.294	9.414	9.465	9.188	9.009
.3898	9.866	10.220	10.191	9.600	9.327	.3858	10.176	10.558	10.485	9.853	9.529	.5328	9.309	9.424	9.478	9.197	9.004
.3940	9.770	10.109	10.047	9.515	9.265	.3964	10.199	10.601	10.496	9.874	9.556	.5350	9.306	9.438	9.482	9.205	9.013
.3975	9.656	10.011	9.894	9.450	9.220	.3986	10.200	10.596	10.492	9.919	9.571	.5423	9.336	9.478	9.483	9.222	9.026
.4125	9.524	9.743	9.690	9.353	9.138	.4008	10.191	10.592	—	9.921	9.570	.5445	9.348	9.479	9.522	9.235	9.033
.4162	9.479	9.700	9.687	9.340	9.119	.4121	10.209	10.606	10.496	9.918	9.574	.5467	9.358	9.491	9.527	9.227	9.032
.4201	9.412	9.637	9.620	9.293	9.099	.4142	10.212	10.601	10.523	9.919	9.581	.5489	9.373	9.512	9.541	9.244	9.035
.4238	9.358	9.561	9.553	9.261	9.073	.4164	10.218	10.614	10.533	9.898	9.582	.5510	9.376	9.523	9.542	9.241	9.028
.4273	9.325	9.509	9.499	9.218	9.022	.4272	10.198	10.575	10.505	—	—	.5612	9.416	9.582	9.598	9.271	9.059
.4418	9.254	9.335	9.424	9.132	8.945	.4624	9.951	10.282	10.167	9.685	9.414	.5634	9.422	9.588	9.649	9.270	9.058
.4454	9.232	9.352	9.435	9.169	8.964	.4646	9.907	10.230	10.125	9.657	9.390	.5656	9.428	9.597	9.622	9.281	9.071
.4490	9.251	9.342	9.417	9.159	8.956	.4668	9.860	10.178	10.043	9.606	9.349	2 451 681+,	-.0003,	-.0011,	.0002,	.0004	
.4528	9.245	9.334	9.419	9.167	8.974	.4799	9.595	9.823	9.727	9.421	9.190	.3336	9.728	10.076	10.184	9.485	9.208
2 449 918+,	-.0007,	-.0028,	.0006,	.0010		.4821	9.580	9.803	9.716	9.397	9.183	.3358	9.726	10.067	10.138	9.490	9.211
.3996	10.220	10.662	10.612	9.887	9.572	.4843	9.565	9.780	9.706	9.402	9.172	.3379	9.732	10.081	10.162	9.490	9.201
.4042	10.207	10.596	—	9.871	9.538	.4942	9.443	9.630	9.639	9.317	9.119	.4315	9.908	10.249	10.139	9.610	9.294
.4085	10.206	10.597	10.690	9.862	9.548	.4986	9.419	9.552	9.597	9.303	9.105	.4337	9.904	10.278	10.200	9.623	9.298
.4129	10.156	10.541	10.338	9.836	9.539	.5087	9.271	9.393	9.417	9.198	9.020	.4359	9.909	10.265	10.184	9.629	9.296
.4173	10.113	10.498	10.333	9.821	9.504	.5109	9.262	9.378	9.396	9.194	9.006	.5144	10.020	10.396	10.301	9.692	9.370
.4337	9.917	10.251	10.162	—	—	.5131	9.255	9.372	9.393	9.179	8.999	.5165	10.019	10.388	10.304	9.702	9.376
.4425	9.660	9.951	9.808	9.441	9.175	.5235	9.225	9.330	9.360	9.147	8.959	.5187	10.033	10.415	10.292	9.702	9.375
.4469	9.598	9.832	9.693	9.400	9.161	.5257	9.227	9.334	9.354	9.163	8.973	.5687	10.043	10.437	10.351	9.735	9.408
.4723	9.328	9.463	9.414	9.180	9.003	.5279	9.230	9.324	9.363	9.142	8.968	.5708	10.048	10.472	10.347	9.731	9.396
.4767	9.248	9.402	9.386	9.134	8.987	.5301	9.235	9.340	9.356	9.154	8.978	.5730	10.051	10.449	10.356	9.748	9.405
2 449 919+,	-.0005,	-.0014,	.0005,	.0008		.5322	9.236	9.348	9.366	9.154	8.982	.5752	10.045	10.453	10.337	9.743	9.408
.3548	9.819	10.159	10.161	9.529	9.209	.5426	9.268	9.392	9.413	9.150	8.982	.5774	10.047	10.450	10.338	9.752	9.406
.3579	9.825	10.179	—	9.510	9.214	.5447	9.269	9.397	9.421	9.164	8.990	2 451 682+,	-.0003,	-.0011,	.0002,	.0004	
.3603	9.830	10.206	10.220	9.558	9.233	.5469	9.273	9.421	9.426	9.189	8.997	.3229	10.120	10.499	10.408	9.800	—
.3628	9.816	10.208	10.182	9.496	9.223	.5491	9.281	9.392	9.446	9.188	8.993	.3251	10.130	10.521	10.430	9.833	—
.3652	9.847	10.185	10.186	9.542	9.214	.5513	9.288	9.421	9.461	9.188	9.000	.3273	10.134	10.511	10.391	9.833	—
.3782	9.851	10.214	10.162	9.560	9.254	.5615	9.325	9.481	9.517	9.249	9.014	.3334	10.154	10.516	10.437	9.846	—
.3807	9.872	10.217	10.276	9.556	9.254	.5636	9.340	9.491	9.517	9.246	9.023	.3356	10.151	10.537	10.428	9.836	—
.3836	9.838	10.203	10.190	9.534	9.217	.5658	9.345	9.500	9.555	9.258	9.030	.3378	10.157	10.537	10.440	9.845	9.501
.3861	9.820	10.199	10.185	9.526	9.246	.5680	9.355	9.523	9.551	9.260	9.028	.3444	10.162	10.564	10.476	9.874	9.542
.3884	9.855	10.193	10.211	9.557	9.255	.5702	9.363	9.528	9.560	9.273	9.031	.3465	10.166	10.559	10.467	9.863	9.542
.4992	10.023	10.436	10.468	9.715	9.385	2 451 680+,	-.0003,	-.0011,	.0002,	.0004		.3487	10.183	10.575	10.525	9.876	9.537
.5019	10.022	10.440	10.418	9.714	—	.3599	10.168	10.558	10.498	9.862	9.524	.3545	10.193	10.591	10.524	9.879	9.540
.5056	10.014	10.430	10.424	9.716	9.379	.3686	10.177	10.571	10.460	9.870	9.549	.3566	10.194	10.611	10.527	9.889	9.560
.5083	10.045	10.462	10.457	9.695	9.371	.3708	10.177	10.582	10.485	9.874	9.549	.3588	10.197	10.594	10.506	9.882	9.549
.5111	10.047	10.472	10.410	9.731	9.390	.3730	10.189	10.576	10.509	9.881	9.557	.3658	10.208	10.623	10.535	9.907	9.570
.5228	10.074	10.504	10.460	9.742	9.405	.3799	10.205	10.599	10.505	9.892	9.575	.3679	10.208	10.607	10.508	9.895	9.575
.5255	10.078	10.483	10.463	9.732	9.411	.3821	10.206	10.612	10.527	9.900	9.567	.3701	10.213	10.586	10.446	9.894	9.565
.5288	10.070	10.510	10.401	9.766	9.418	.3842	10.215	10.615	10.517	9.904	9.569	.3759	10.208	10.605	10.565	9.910	9.586
.5315	10.076	10.478	10.380	9.736	9.403	.3916	10.216	10.626	10.533	9.905	9.573	.3781	10.227	10.630	10.550	9.914	9.588
.5342	10.077	10.520	10.398	9.747	9.416	.3937	10.228	10.618	10.543	9.915	9.575	.3803	10.231	10.632	10.575	9.906	9.573
.5455	10.070	10.479	10.408	9.719	9.416	.3959	10.221	10.618	10.494	9.907	9.575	.3877	10.215	10.608	10.521	9.918	9.579
.5481	10.055	10.488	10.410	9.773	9.429	.4025	10.232	10.626	10.501	9.918	9.591	.3899	10.218	10.601	10.503	9.894	9.571
.5507	10.075	10.524	10.446	9.750	9.409	.4046	10.227	10.607	10.537	9.926	9.585	.3920	10.217	10.564	10.526	9.910	9.583
.5532	10.093	10.438	10.365	9.749	9.421	.4068	10.219	10.630	10.511	9.920	9.593	.3981	10.203	10.581	10.507	9.901	9.576
.5558	10.071	10.503	10.499	9.763	9.418	.4145	10.207	10.602	10.483	9.903	9.582	.4003	10.195	10.565	10.517	9.894	9.578
2 449 921+,	-.0006,	-.0013,	.0008,	.0011		.4166	10.197	10.592	10.451	9.892	9.566	.4025	10.188	10.563	10.508	9.893	9.573
.3402	9.851	10.204	10.223	9.578	9.279	.4188	10.182	10.579	10.475	9.894	9.573	.4101	10.143	10.523	10.428	9.859	9.558
.3432	9.857	10.211	10.242	9.557	9.270	.4338	10.077	10.462	10.345	9.810	9.507	.4122	10.125	10.496	10.405	9.839	9.519
.3462	9.863	10.240	10.248	9.575	9.284	.4359	10.049	10.430	10.336	9.785	9.483	.4144	10.102	10.470	10.375	9.815	9.523
.3493	9.881	10.240	10.253	9.594	9.291	.4381	10.029	10.381	10.304	9.752	9.460	.4202	10.023	10.379	10.306	9.761	9.476
.3524	9.886	10.253	10.278	9.605	9.284	.4457	9.913	10.252	10.176	9.662	9.387	.4224	9.992	10.325	10.285	9.736	9.445
2 450 939+,	-.0018,	-.0058,	.0009,	.0014		.4478	9.862	10.193	10.082	9.625	9.350	.4246	9.954	10.287	10.216		

and the length  $l$  of the light curve was computed by

$$\nu l(P, \boldsymbol{\delta}) = \{(\phi_{N_V} - \phi_1)^2 + [V(\phi_{N_V}) - V(\phi_1)]^2\}^{1/2} + \sum_{j=2}^{N_V} \{(\phi_{j-1} - \phi_j)^2 + [V(\phi_{j-1}) - V(\phi_j)]^2\}^{1/2}, \quad (7)$$

$\nu$  is a normalization factor.

- (3) The minimum of  $l(P, \mathbf{0}) = l_0$  was determined as a function of the trial period  $P$ .
- (4) By a variation of  $\delta_k$ ,  $k = 1, \dots$  the absolute minimum of  $l(P, \boldsymbol{\delta} \neq \mathbf{0}) < l_0$  was determined.

If the period is changing and it can be expanded around the point  $t'_0$ ,

$$P(t) = P^{(0)} + P^{(1)}(t - t'_0) + \frac{1}{2}P^{(2)}(t - t'_0)^2 + \dots, \quad (8)$$

the phase shift will have a systematic component

$$\delta_k^{\text{sys}} = \int_{t'_0}^{t_k} [P^{(1)}(t - t'_0) + \dots] dt. \quad (9)$$

$\delta_k = \delta_k^{\text{sys}} + \delta_k^{\text{noise}}$  defines the phase noise  $\delta_k^{\text{noise}}/P^{(0)}$ ,  $\delta_k^{\text{sys}}$  is a clearly defined quantity by (9) in contrast to  $\delta_k^{\text{noise}}$  which can indicate a real phase shift or from cycle to cycle small systematic differences in the shape of the light curve.

An observed magnitude  $V$  is composed of the brightness of the star and a random error. The above procedure isolates the brightness variations  $V_P(\phi_j)$  of period  $P(t)$  while those of different period(s)  $V_{\bar{P}}(\phi_j)$  and the random observational error  $s(\phi_j)$  manifest themselves as a scatter i.e.  $V(\phi_j) = V_P(\phi_j) + V_{\bar{P}}(\phi_j) + s(\phi_j)$ . The average random component  $\bar{s}_r$  from  $s(\phi_j)$  is the same for any  $P$  and it can be extracted from the observation of the comparison star(s):  $\bar{s}_r \approx \sigma_V$

In points 3.1.1–3.1.4 the introduction of  $\boldsymbol{\delta}$  is the surplus to SLM of Dworetzky (1983). In addition to it some useful quantities and relations will be derived purely from the observational data if  $N_V$  is sufficiently large i.e.  $2(V_{\min} - V_{\max})/N_V \ll \bar{s}_r$ ,  $V(\phi)$  is monotonic between maximum and minimum, the distribution of  $\phi_j$  is uniform, and the average phase difference  $\Delta\phi = \overline{\phi_j - \phi_{j-1}} \approx 1/N_V$  vanishes. These conditions will be denoted by  $(\alpha_1 - \alpha_4)$ .

(a) *The scatter along the light curve* is roughly

$$\bar{s}(P, \boldsymbol{\delta}) = \left[ \sum_j \{ [V(\phi_{j-1}) - V(\phi_j)]/2 \}^2 / (N_V - 1) \right]^{1/2}, \quad (10)$$

$[V(\phi_{j-1}) + V(\phi_j)]/2$  was taken as average for any  $j$ , the difference of the average and  $V(\phi_j)$  was taken as scatter at  $\phi_j$ . With these conventions  $\bar{s}$  is the phase dispersion (Stellingwerf 1978) for the data which were not binned or the  $\theta$  of Stetson (1996) with a different normalization.

(b) *At a wrong period* the saw-tooth function produces random distribution  $V(\phi_j)$ , therefore the absolute value of the mean magnitude difference and the scatter will be

$$|\overline{\Delta V(\phi_j)}| \approx (V_{\min} - V_{\max})/2, \quad (11)$$

$$\bar{s} \approx (V_{\min} - V_{\max})/4. \quad (12)$$

From (12)  $\bar{s}$  will be the upper limit of the scatter as a function of  $P$ . For the normalization factor a good choice is

$$\nu = N_V(V_{\min} - V_{\max})/2 \quad (13)$$

by which  $l < 1$  is provided since  $\nu$  is an upper limit for the sum of the string lengths.

(c) *At the true  $P(t)$*   $\Delta V_P$  vanishes for any  $j$  with increasing  $N_V$ :

$$\Delta V_P = |V_P(\phi_{j-1}) - V_P(\phi_j)| \approx 2(V_{\min} - V_{\max})/N_V \rightarrow 0, \quad (14)$$

therefore, the value  $\bar{s}$  will be determined by the scatter  $s(\phi_j) > \Delta V_P$ . If

$$\bar{s}(P, \boldsymbol{\delta}) \approx \sigma_V \quad (15)$$

the time series  $V(t_i)$  has the sole period  $P$  with phase shifts  $\boldsymbol{\delta}/P$ . If (15) is not satisfied at the minimal  $l(P, \boldsymbol{\delta})$  a residual signal of amplitude  $4[\bar{s}(P, \boldsymbol{\delta}) - \sigma_V]$  can be expected as it follows from (11), (12). To find the period of this residual signal the procedure must be repeated after prewhitening  $V(t)$  with period  $P(t)$ . For monoperiodic signal ( $V_{\bar{P}}(\phi_j) \equiv 0$ ) the lower limits  $l = 2[4 + 1/(V_{\min} - V_{\max})^2]^{1/2}/N_V \rightarrow 0$  and  $s = (V_{\min} - V_{\max})/[N_V(N_V - 1)]^{1/2} \rightarrow 0$  are obtained if  $s(\phi_j) = 0$  is assumed for any  $j$  at the true period.

(d) *The error of  $P^{(0)}$*  resulting from random observational error can be estimated as follows. The random errors  $\Delta V(t_i)$  of normal distribution result in random errors  $s(\phi_j)$  of normal distribution characterized by  $\sigma_V$ . To estimate the change of  $l_0$  as a function of  $\sigma_V$  the random errors can be replaced by  $s(\phi_j) = \sigma_V$  for  $N_V/2$  phase points and by  $s(\phi_j) = -\sigma_V$  for the rest. Therefore,  $l_0$  will change roughly as the hypotenuse of a rectangular triangle from  $\Delta\phi$  and  $2\sigma_V$  times average number  $N_V/2$  of these triangles along the light curve:

$$\Delta l_0 \approx \frac{N_V \sigma_V}{2\nu} \frac{\partial}{\partial \sigma_V} (\overline{\Delta\phi}^2 + 4\sigma_V^2)^{1/2} + O(\sigma_V^2) \approx \frac{2\sigma_V}{V_{\min} - V_{\max}}, \quad (16)$$

if  $V_{\bar{P}}(\phi_j) \equiv 0$  is assumed and conditions  $\overline{\Delta\phi} = N_V^{-1} \ll 4\sigma_V$  ( $\alpha_5$ ),  $|V_P(\phi_{j-1}) - V_P(\phi_j)| \ll \sigma_V$  for any  $j$  ( $\alpha_6$ ) are substituted for ( $\alpha_4$ ), ( $\alpha_1$ ) respectively. (The average number of the triangles is obtained by taking into account that the probability is the same for a light curve with steps  $-2\sigma_V$  at  $N_V/2$ ,  $2\sigma_V$  at  $N_V$ , i.e. two triangles, and for a light curve with steps in every point  $j$ , i.e.  $N_V$  triangles etc.). On the other hand

$$\Delta l_0 = \frac{1}{2} \frac{\partial^2 l_0}{\partial P^{(0)2}} \Delta P^{(0)2} \quad (17)$$

because  $\partial l_0 / \partial P^{(0)} = 0$  at a true  $P^{(0)}$ .  $\partial^2 l_0 / \partial P^{(0)2}$  can be obtained from fitting a polynomial to  $l_0(P^{(0)})$  at  $P^{(0)}$ , the narrower the minimum the smaller error  $\Delta P^{(0)}$  will be the result from

$$\Delta P^{(0)} \approx \left[ 4\sigma_V / (V_{\min} - V_{\max}) \frac{\partial^2 l_0}{\partial P^{(0)2}} \right]^{1/2}. \quad (18)$$

If for a time interval covering epochs of number  $E$  the values  $\delta_k$  are known to a  $P^{(0)}$  and for the maxima  $\text{HJD}_k = \text{HJD}_0 + P^{(0)}E_k + \delta_k$  can be assumed with a normal distribution of  $\delta_k$  characterized by the SD  $\sigma_{\delta_k}$  the error of  $\Delta P^{(0)}$  can be estimated from

$$\Delta P^{(0)} \approx 2\sigma_{\delta_k}/E. \quad (19)$$

This estimation will be used for old observations since the epochs of maxima were given only.

(e) *The error of a  $\delta_k$*  can be estimated similarly. Assuming an averaged  $\sigma_V$  perturbation on the  $k$ th segment of a smooth light curve will change  $l$  roughly by

$$\Delta l \approx 2n\sigma_V \frac{\partial}{\partial \sigma_V} \left( \overline{\Delta\phi^2} + \sigma_V^2 \right)^{1/2} / \nu \approx 2n\sigma_V / \nu \quad (20)$$

if the appropriate conditions  $\alpha$  are satisfied and  $n \ll N_V$ . On the other hand

$$\Delta l = \frac{1}{2} \frac{\partial^2 l_0}{\partial \delta_k^2} \Delta \delta_k^2 \quad (21)$$

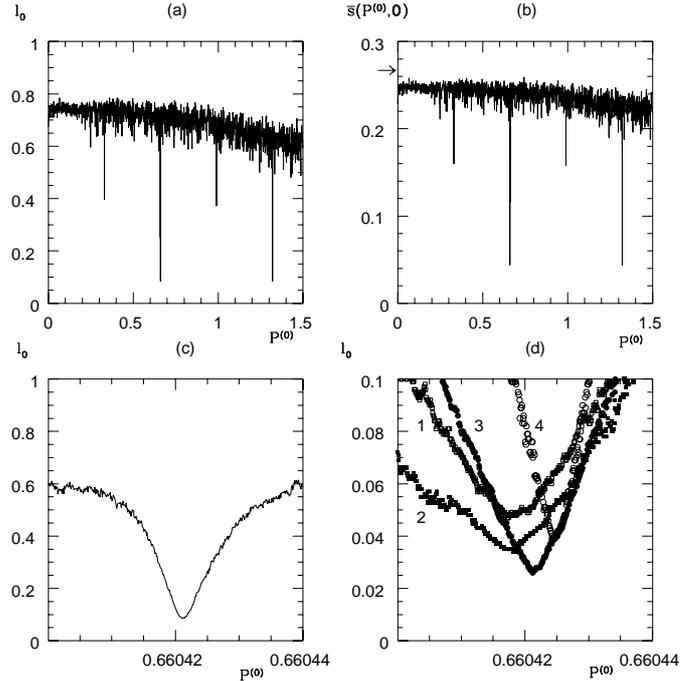
because  $\partial l / \partial \delta_k = 0$  at the minimum from which

$$\Delta \delta_k \approx \left[ 4n\sigma_V / N_V (V_{\min} - V_{\max}) \frac{\partial^2 l}{\partial \delta_k^2} \right]^{1/2}, \quad (22)$$

$\partial^2 l / \partial \delta_k^2$  can be obtained from fitting a polynomial to  $l(\delta_k)$  at the minimum. The interpretation of this error is more problematic because two different effects are non-separably mixed in  $\delta_k^{\text{noise}}$ : real phase noise and eventual non-repetitiveness of the light curve.

SLM is the mathematical formulation of what a smooth light curve is after folding the observed magnitudes. Its advantages are simple computational realization, objective criterion (15) indicates whether all information content of the observed magnitudes was exploited, a priori information on the light curve (e.g. shape, existence of a rapidly converging Fourier series to it) were not necessary, in spite of this the error of the period and phase noise could be estimated. It is remarkable that the error estimations (18), (22) are insensitive to the shape of the light curve. Conditions ( $\alpha$ ) must only be satisfied. Furthermore conditions ( $\alpha$ ) permit to plan observations or to decide whether the available observational material is or is not sufficient to draw some conclusions because by using (10)–(22) relations were provided among string length, observed amplitude, observational error, number of observations, and monop periodicity or error of the derived quantities period and phase noise.

When O–C curve is constructed for a period  $P$  the usual procedure is to select maxima or a fixed phase of the light curve. A small part of the observational material is used: a few points around the selected phase e.g. for a parabolic fit and this is a source of some error. Here all observed magnitudes are of the same rank in the period search and the analogue  $\delta_k$  of O–C is provided for every light curve segment irrespective of its being or not being part of the selected phase. The  $V$  file contains 19 segments with maxima i.e. conventional O–C values of number 19



**Fig. 1.** Panels **a–c**) are a plot of (7), (10) using all  $V$  observations (1441 points). The arrow in Panel **b**) indicates  $(V_{\min} - V_{\max})/4$  according to (12). Panel **d**) is a zoomed plot of  $l_0$  for the subgroups, its curves are labeled according to the notation in Table 6, the shift of the period  $0^{\text{d}}.6604173 \rightarrow 0^{\text{d}}.6604244$  is clearly visible.

would be available while the number of the  $\delta_k$  values will be 49.

The link to PDM is that a minimum of  $l$  is equivalent to a minimum of  $\bar{s}$  if conditions ( $\alpha$ ) hold. Minimizing  $l$  is better because binning of the folded magnitudes and their averaging to determine the scatter are not necessary, furthermore  $l$  is a more smooth function than  $\bar{s}$ , especially if the data are noisy and there are uncovered intervals on the axis  $\phi$  which are interpolated by straight lines in SLM.

### 3.2. Results from the $V$ data

The data set  $V$  satisfies conditions ( $\alpha$ ).

The observations were divided in 4 subgroups, Table 6 gives the periods for them and for the whole material which were determined according to the previous subsection. Figure 1 is a plot of the numerical results. In Figs. 1a,b the deepest minima correspond to  $P$  and  $2P$  while less deep minima can be seen at  $0.5P$ ,  $1.5P$ . Figure 1d demonstrates clearly that the period is increasing. For check the PDM routine of IRAF was used for the full data set. The difference of the periods is  $0^{\text{d}}.0000012$ , which indicates the error of 3.1.1–3.1.4 or PDM for these data, this value agrees well with  $\Delta P^{(0)}$  from (18), parabola was fitted to  $l_0(P^{(0)})$ ,  $\sigma_V = 0.015$  mag was used for 1–5 in Table 6. (In the interval  $0^{\text{d}}.660420 < P^{(0)} < 0^{\text{d}}.660423$  the curves  $l_0$  and  $\theta$  of PDM have both shallow minimum.)

**Table 6.** The periods from minimizing (7) and their errors in units day.

JD–2 400 000	$N_V$	$P^{(0)}$	$\Delta P^{(0)}$	$l_0$	$\bar{s}(P^{(0)}, \mathbf{0})$
36152 – 37045 <sup>1</sup>	454	.6604173	$\pm .0000091$	.047	.018
42403 – 43204 <sup>2</sup>	318	.6604183	$\pm .0000094$	.034	.016
44989 – 47228 <sup>3</sup>	441	.6604211	$\pm .0000032$	.026	.009
49817 – 51682 <sup>4</sup>	228	.6604244	$\pm .0000034$	.039	.018
36152 – 51682 <sup>5</sup>	1441	.6604211	$\pm .0000011$	.086	.047
36152 – 51682 <sup>6</sup>	1441	.6604223	—	.117	.062

<sup>1</sup> Oláh & Szeidl (1978) plus Spinrad (1961).

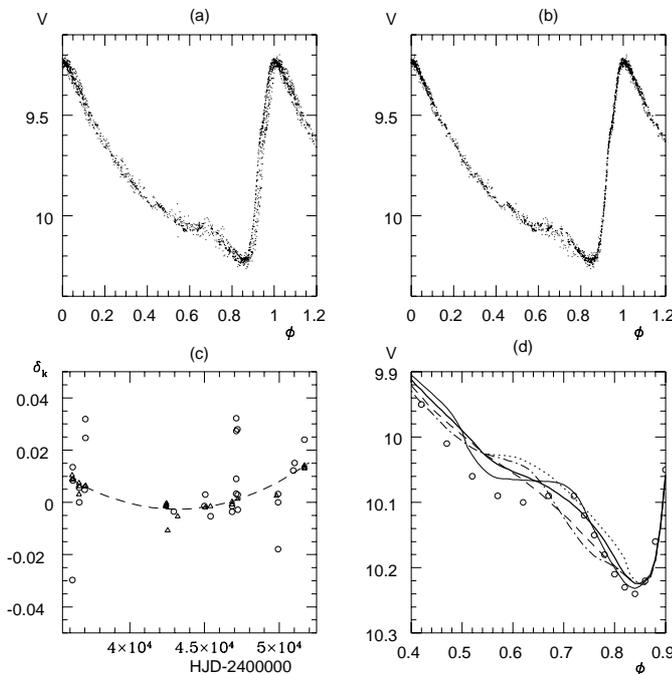
<sup>2</sup> Oláh & Szeidl (1978).

<sup>3</sup> Present observations plus Liu & Janes (1989).

<sup>4</sup> Present observations.

<sup>5</sup> Sum of 1–4.

<sup>6</sup> Sum of 1–4, period from PDM, not from minimizing (7).



**Fig. 2.** Panel **a**) is a plot of the folded light curve with  $P = 0^d.6604211$ ,  $\delta = \mathbf{0}$  ( $l_0 = 0.076$ ,  $\bar{s} = 0.047$  mag), in Panel **b**) applying  $\delta$  of Panel **c**) resulted in  $l = 0.027$ ,  $\bar{s} = 0.010$  mag. In Panel **c**) the values  $\delta_k$  from ascending and descending branch are denoted by triangles and circles, respectively. The dashed line is a weighted parabolic fit to them. In Panel **d**) fitting curves are plotted, thick line: averaged light curve to  $k = 1-51$ , dotted:  $k = 15, 16$ , dashed:  $k = 2, 4, 8, 10$ , dot-dashed:  $k = 31, 34-41$ , thin line:  $k = 25, 27-29, 42-44, 46-50$ , circles: computed from Oke et al. (1962).

The folded light curve with the average period  $P^{(0)} = 0^d.6604211$  is plotted in Fig. 2a, it has a scatter  $\bar{s}(0.6604211, \mathbf{0}) = 0.047$  mag  $\approx 3\sigma_V$ . The folded light curve with the period from PDM is similar, however, it is wider corresponding to  $\bar{s}(0.6604223, \mathbf{0}) = 0.062$  mag.

The values  $\delta_k$  were determined iteratively in the following sequence  $l(P^{(0)}, \mathbf{0}) > l(P^{(0)}, \delta_1^{(1)}, 0, \dots) > l(P^{(0)}, \delta_1^{(1)}, \delta_2^{(1)}, 0, \dots), \dots, l(P^{(0)}, \delta_1^{(2)}, \delta_2^{(1)}, \dots), \dots$  until having reached less change than  $0^d.0001$  in any component of  $\delta$ , the upper indices (1), (2) indicate the serial number of the iterative step, seven steps satisfied the accuracy requirement. One of  $\delta_k$  was of course arbitrary. With parabolic fits and  $\sigma_V = 0.015$  mag from (22) the error estimation of  $\delta_k$  gave roughly  $0^d.003$  for ascending branch while for descending branch  $0^d.03$  was the typical value. Greater  $n$  gave smaller  $\Delta\delta_k$ , e.g.  $\Delta\delta_{25} = 0^d.0067$  was found for this descending branch with  $n = 80$ .

If the  $\delta_k$  values of Fig. 2c were applied the scatter decreased to  $\bar{s}(0.6604211, \delta) = 0.010$  mag and the corresponding light curve in Fig. 2b became remarkably narrower except for the phase interval  $0.4 \leq \phi \leq 0.85$ . Because of the length of the segments an arbitrary phasing for  $0.4 \leq \phi \leq 0.85$  could even result in a somewhat narrower curve, however, this would deteriorate the narrowness outside this phase interval.

Omitting the point segment  $k = 12$ , merging  $k = 41$  into  $k = 40$ , and weighting the  $\delta_k$  values by the number  $n$  of the points in the  $k$ th light curve segment the quadratic fit

$$\delta_k = 0.4078 - 1.899 \times 10^{-5}(t - t'_0) + 2.199 \times 10^{-10}(t - t'_0)^2 \quad (23)$$

was found with a SD  $0^d.0065$ ,  $t'_0 = 2443181$ , this SD is in accordance with the estimation of  $\Delta\delta_k$  from (22). In Fig. 2c (23) is plotted by dashed line, the circles and triangles represent the  $\delta_k$  values from descending and ascending branch respectively. Wunder (1991) gave the maxima HJD = 2448024.4056, 2448122.4237.  $\delta = -0^d.0071$ ,  $-0^d.0127$  belong to them, however, they were not included in the regression because of lacking data for their weight and reality.

It is remarkable that the outlier points  $\delta_2 = -0^d.0297$ ,  $\delta_{15} = 0^d.0319$ ,  $\delta_{17} = 0^d.0247$ ,  $\delta_{36} = 0^d.0273$ ,  $\delta_{37} = 0^d.0322$ ,  $\delta_{38} = 0^d.0280$ ,  $\delta_{45} = -0^d.0179$  come all from a descending branch segment. They consist of  $n = 5, 30, 5, 4, 6, 8, 5$  points with centre of phase  $\phi = 0.564, 0.632, 0.273, 0.557, 0.647, 0.746, 0.312$ ,  $\Delta\delta_k \approx 0^d.03$  was found for them. In the descending branch the slope of the light curve is smaller which can lead to more uncertain  $\delta_k$  values as it is suggested by the values from (22), however, these large phase shifts cannot be considered as mere observational or random error. Strict or loose phasing are improbable when the pulsation cycle is just in ascending or descending branch, respectively. To clarify the suspected light curve variations in the phase interval  $0.4 \leq \phi \leq 0.9$  cubic curves were fitted to the  $V$  observations from different epochs. These and the  $V$  light curve from the spectrophotometry (Oke et al. 1962) are plotted in Fig. 2d. The segregation of the different curves exceeds significantly the acceptable random scatter  $\sigma_V$  and it is responsible for the larger scatter of the points in Fig. 2b around  $\phi \approx 0.6$ .

From (23) the changing period is represented by

$$P(t) = 0.6604211 \pm 0.0000011 \\ + (4.40 \pm .14) \times 10^{-10}(t - 2443181) \quad (24)$$

leading to  $P(2452000) - P(2436200) = (6.9 \pm .2) \times 10^{-6}$  days in accordance with the values reported in Table 6. Finally (24) was introduced in (9) and (7): the result is  $l(P(t), \mathbf{0}) = .040$  with  $\bar{s}(P(t), \mathbf{0}) = 0.014$  mag. The data in Table 6 suggest a quadratic term to (24) and (9), however, a cubic fit to  $\delta_k$  leading to  $P^{(2)}$  in (24) did not reduce  $\bar{s}(P(t), \mathbf{0})$  i.e. the information content of the  $V$  data is represented fully by (23) and (24). If  $\sigma_V = 0.015$  mag is accepted it can be stated from the available observations definitely that SU Dra has a single period which varies according to (24), furthermore up to 20–30 min/ $P^{(0)}$  phase noise or rather systematic magnitude differences of a few 0.01 mag are present before the ascending branch.

A residual variation on a few 10 mmag level cannot be excluded according to (12) since  $\bar{s}(0.6604211, \delta) = 0.010$  mag could be achieved. To discriminate this eventual systematic residual signal from a phase noise the period search was repeated after subtracting  $V_{P(t)}(\phi)$  with (24) from the  $V$  data. Because of the non-uniform distribution of  $\phi_j$  the attempts to reproduce  $V_{P(t)}(\phi)$  by a single function for  $0 \leq \phi \leq 1$  have failed: e.g. a Fourier series with 25 terms had a scatter 0.016 mag, but it deviated from the observed points systematically at the hump ( $\phi \approx 0.92$ ), and this remained true if the input of the Fourier analysis was a smoothed light curve with uniform distribution of  $\phi$ . Therefore, the subtraction of the average light curve was made by fitting cubic polynomials to  $V_{P(t)}(\phi)$  in 9 intervals  $\phi$ . This way of prewhitening the signal was found optimal, it corresponds to the conventional way, e.g. Templeton (1997), but it fits better to the distribution of  $\phi_j$  and shape of the light curve. The statistical characteristics of the residual  $V$  data are: the magnitude averaged mean is 0.001 mag with SD 0.020 mag and skewness  $-0.225$ , the maximum and minimum are 0.086 mag and  $-0.108$  mag, respectively. First it was checked that the subtraction extirpated  $V_{P(t)}(\phi)$  with (24) from the residual data indeed. The period search has not revealed further period in the interval  $0.001 \leq P^{(0)} \leq 1200$ .

The conventional form for the epoch of the  $E$ th maximum is

$$\text{HJD}_{\max} = 2443181.5280 + 0.6604211 E \\ + 9.56 \times 10^{-11} E^2, \quad (25)$$

when calculating  $\delta_k^{\text{sys}} t_k = P^{(0)} E$  was used in (9) which causes a negligible error for some 100 years.

### 3.3. Mean light and colour curves

The application of  $\delta$  for the  $U, B, R_C, I_C$  data reduced  $\bar{s}(0.6604211, \mathbf{0}) = 0.050, 0.047, 0.031, 0.023$  mag to  $\bar{s}(0.6604211, \delta) = 0.037, 0.020, 0.013, 0.013$  mag respectively. From the latter points averaged light curves were

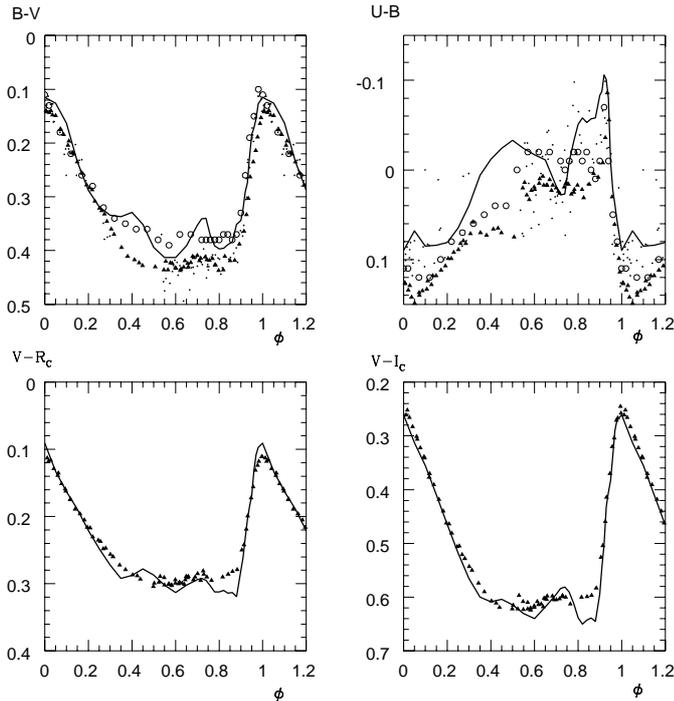
**Table 7.** The averaged light curves.

$\phi$	$V$	$B$	$U$	$R_C$	$I_C$
0.1	9.233	9.348	9.438	9.142	8.972
0.05	9.313	9.439	9.507	9.180	9.000
0.1	9.435	9.598	9.683	9.272	9.080
0.15	9.538	9.772	9.856	9.349	9.128
0.2	9.635	9.923	10.004	9.414	9.170
0.25	9.726	10.045	10.110	9.478	9.208
0.3	9.796	10.130	10.170	9.524	9.231
0.35	9.860	10.197	10.203	9.568	9.260
0.4	9.912	10.241	10.229	9.624	9.303
0.45	9.952	10.303	10.279	9.674	9.348
0.5	9.990	10.384	10.351	9.703	9.376
0.55	10.027	10.440	10.416	9.725	9.397
0.6	10.053	10.466	10.449	9.740	9.413
0.65	10.069	10.460	10.449	9.767	9.451
0.7	10.093	10.446	10.464	9.799	9.498
0.72	10.105	10.446	10.474	9.813	9.521
0.74	10.120	10.460	10.487	9.825	9.539
0.76	10.138	10.510	10.501	9.835	9.548
0.78	10.159	10.550	10.518	9.847	9.547
0.8	10.184	10.581	10.530	9.872	9.546
0.82	10.210	10.606	10.548	9.900	9.560
0.84	10.224	10.613	10.560	9.910	9.581
0.86	10.219	10.603	10.546	9.906	9.580
0.88	10.186	10.537	10.479	9.867	9.541
0.9	10.061	10.405	10.322	9.781	9.467
0.91	9.957	10.279	10.191	9.696	9.413
0.92	9.825	10.116	10.010	9.593	9.323
0.93	9.670	9.944	9.843	9.465	9.238
0.94	9.589	9.814	9.734	9.404	9.184
0.95	9.535	9.719	9.710	9.366	9.152
0.96	9.446	9.618	9.640	9.314	9.108
0.97	9.355	9.510	9.549	9.247	9.065
0.98	9.290	9.417	9.474	9.193	9.020

constructed for these bands by cubic spline functions, the result is given in Table 7.

The  $B$  and  $U$  light curves of Oláh & Szeidl (1978) were omitted from the analysis. However, by the zero point shift  $-0.08$  mag a similar shape of their  $B$  light curve segments was found as in Fig. 2d: straight lines below the averaged curve. After a zero point shift of  $\approx 0.13$  mag their  $U$  curves, in spite of a larger scatter, show approximate congruence around the ascending branch and the segregation is clearly seen in the phase interval  $0.4 \leq \phi \leq 0.85$ .

Figure 3 is a plot of the colour curves. A comparison of the Spinrad (1961), Oke et al. (1962), Liu & Janes (1989), and present light curves shows that the amount of segregation is increasing toward shorter wavelengths: from 0.05 mag in  $V$  to 0.1 mag in  $B$  and 0.2 mag in  $U$ . Because of the uncertainties with the  $U$  band a part of the segregation may originate from the photometric realization of  $U$ . It is remarkable that outside the interval  $0.4 \leq \phi \leq 0.85$  the light and colour curves coincide well in  $B, V$ , and the coincidence is quite satisfactory if  $U$  is considered. The sharp peak  $U - B \approx -0.11$  is real at  $\phi = 0.92$ : it is clearly visible in each colour curve segments of the present observations and Liu & Janes (1989). The large integration time of Oke et al. (1962) smoothed it out to  $U - B \approx -0.07$ . The  $U - B$  points of Spinrad (1961) were plotted rather for the sake of completeness, in the further considerations their weight will be small because of the large scatter. The number of the observations  $R_C, R$  and  $I_C, I$  is not sufficient to draw definite conclusion whether do or do not segregate the light curve segments in the phase interval  $0.4 \leq \phi \leq 0.85$ , nevertheless a yes seems plausible.

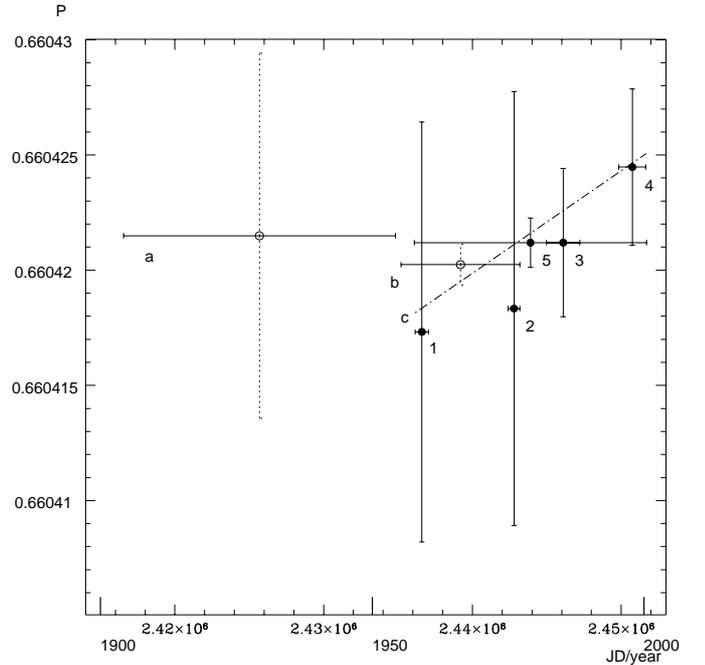


**Fig. 3.** The colour curves. Thick line: averaged colour curves, the non-significant structures (peak-like variations  $<0.03$  mag in neighbouring phase points) were not smoothed out. Circles: computed from Oke et al. (1962), triangles: Liu & Janes (1989), dots: Spinrad (1961).

#### 4. Discussion

Using *Hipparcos* kinematic data (ESA 1997) and assuming a distance  $\approx 350$  pc the perspective acceleration of SU Dra was calculated, it causes  $O(10^{-12})$  day/day period change, thus, the value given in (24) is intrinsic to SU Dra. The conclusion of Oláh & Szeidl (1978) was that the available photoelectric observations from 1957 to 1977 could be accounted for by the constant period  $P^{(0)} = 0^d.66042001$ . For this interval a linear regression to their Table 3 gave  $P^{(0)} = 0^d.6604202 \pm 0^d.0000009$  while for the previous photographic and visual observations from 1904 to 1955  $P^{(0)} = 0^d.6604214 \pm 0^d.0000079$  was obtained, the errors were estimated from (19). Their O–C curve (to period  $0^d.66041890$ ) has a wavy structure with amplitude  $\approx 0^d.01$  and quasi-period  $\approx 20$  years. The present study has demonstrated that the period was increasing monotonically for a longer time than twice the length of this quasi-period. The data in Table 6 have suggested that the period change could be composed from two linear parts accelerating (perhaps rapidly):  $P^{(1)} = 1.7 \times 10^{-10}$  (points 1–2)  $\rightarrow 7.8 \times 10^{-10}$  day/day (points 2–4). The period of the interval 1904–1955 is above the period for 1957–1960, 1974–1977 reported in Table 6: it suggests that a monotonous  $P^{(1)} > 0$  for 1904–2000 is improbable. Figure 4 summarizes the results on period.

From the error bars two conclusions can be drawn. Longer intervals even with noisy data or using two comparison stars (in subgroups 3, 4 via less noisy data)



**Fig. 4.** The period of SU Dra. The filled circles with error bar are labelled according to Table 6. **a)** Oláh & Szeidl (1978) from visual and photographic observations, **b)** Oláh & Szeidl (1978) from photoelectric observations only, the periods and the error bars were determined from a linear regression to their Table 3, its SD was introduced as  $\sigma_{\delta_k}$  in (19). **c)** Eq. (24).

resulted in smaller error  $\Delta P^{(0)}$ . The dotted and solid error bars characterize different features: how coherent sets of signals are the maxima and the whole light curve, respectively. The maxima are recurring more stably.

Since SU Dra is a metal deficient star with Population II kinematics for estimating the evolutionary period change the models of Lee (1991) were used. According to them it would be accelerating rapidly and monotonically to some  $10^{-9}$  day/day in the final phase, at an age  $\approx 8 \times 10^7$  years from the zero age horizontal branch. The empirical  $P^{(1)}$  of the present study is in accordance with the theoretical value, but the slope is not monotonous between 1904 and 2000 and it is one order of magnitude greater than the computed theoretical slope. Therefore, the random fluctuation in the stellar structure (Schweigart & Renzini 1979; Cox 1998) is a plausible explanation for the period change.

The presence of the light curve segregation in the phases  $0.4 \leq \phi \leq 0.85$  and its lack outside this interval are convincing evidence that the segregation is a real phenomenon which cannot be explained by an eventually non-perfect realization of the photometric system. In a Baade-Wesselink analysis it is tacitly assumed that the atmosphere is stable in the descending branch, being in the state of free fall which produces repetitive broad-band light curves. The results of this study have shadowed this generally accepted view and suggested that the hydrodynamical behaviour of this stellar atmosphere is more

complicated than that of a spherical pulsation with strict periodicity. Some variation amounting to 0.05 mag has been found even in  $V$  and higher values towards ultraviolet. This variation is present before and at the bump phase when the early shock (Gillet & Crowe 1988) hits the atmosphere. For the sake of completeness it must be mentioned that in the descending branch short term variations were observed photographically by Martin & Plummer (1913) which have never recurred (Oláh & Szeidl 1978). The visual light curves observed by Sperra (1909) originated perhaps from similar light curve segregation in the bump phase which were reported in this paper. However, the error of these early observations is larger than the segregation.

A survey of published light curves revealed similar segregation in the case of SS For (Cacciari et al. 1989; Cacciari et al. 1987) and W Crt (Skillen et al. 1993a; Skillen et al. 1993b). Concerning these stars the phenomenon was registered without detailed discussion, perhaps because of the lack of sufficient observational material: 100 and 129 multicolour observations were obtained, respectively. This could not have made sure the observers whether a sporadic or repetitive feature of the light curve was observed. These stars have steep ascending branch as well as SU Dra. SS For has almost the same metal deficiency  $[\text{Fe}/\text{H}] = -1.5$  while W Crt is less metal deficient,  $[\text{Fe}/\text{H}] = -0.7$ . Common is in all three stars that the amplitude was not variable but the bump was occasionally missing i.e. at some epochs the light curve was similar to the dashed straight line in Fig. 2d both in  $B$  and  $V$  while at other epochs the bump existed, what is more in variable form. The present study has called the attention to the feature in the case of SU Dra and revealed that it may be a common phenomenon among  $\text{RR}_{\text{ab}}$  stars with steep rising branch but could not report eventual periodicity if there is any. The latter negative conclusion has been caused by the sparse coverage of the critical phases. The observed constant amplitude of the full light curve shows that the variation is not Blazhko-effect.

The variations in photometric bands  $V, B, U$  reveal the physical state of the increasingly higher atmospheric layers. From the present study observational evidence has arisen for the instability of these layers from cycle to cycle during the pulsations. Understanding these processes will be important not only for their own right but it will have an effect on the distance calibration of RR Lyrae stars. A forthcoming paper will be devoted to this problem.

## 5. Conclusions

For an analysis in terms of atmospheric models accurate averaged light curves of SU Dra have been compiled in  $UBV(RI)_C$  photometric system using all available observations spanning over 45 years. The observational material has been elaborated by string length minimization hand tailored to slowly changing period, phase noise, and sporadically obtained observations over a long time. Error

estimations have been given by analytical and simple statistical considerations which can be of more general interest since they permit planning observations or reviewing observed data from point of view of completeness: how large error of period, phase shift can be expected from the input data i.e. observed variations, number of observations and their distribution over time, error of an observation characterized by standard deviation. SLM provided the same weight to all observations, for the full data set it gave practically identical period with that of PDM, it has allowed the phasing over time interval 45 years and the derivation of systematic phase changes and the phase noise. This is more than a simple O–C curve since it gave the phase shift to light curve segments containing some 4–5 points at any phase, irrespective whether around maximum or not. The by-products are: the  $V$  light curve is a monophasic but not fully coherent signal, the better determination of the period change, and the averaged light curve itself. The broad band light curves, their non-Blazhko type variability, and the secular behaviour of the period have been summarized in this paper separately since the observational results of the present study form a rather independent entity from the interpretation in terms of atmospheric models and determination of the physical parameters of SU Dra.

*Acknowledgements.* The author wishes to thank L. G. Balázs, J. M. Benkő, A. Holl, Z. Kolláth, B. Szeidl, I. Tóth for numerous help in finding the way in the labyrinth of observational data managing, reduction, RR Lyrae light curves, and J. M. Benkő, Z. Kolláth for comments on the manuscript. Comments of an anonymous referee are gratefully acknowledged.

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