

# Period and amplitude variations in the high-amplitude $\delta$ Scuti star AE Ursae Majoris\*

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**Abstract.** We present a comprehensive investigation of the variations of period and amplitude in the high-amplitude  $\delta$  Scuti star AE UMa based on our new Johnson  $V$  time-series measurements and the existing data. No additional frequencies were detected even though all the available data sets from 1974 to 2001 were analysed. The light variations of AE UMa can be well-reproduced with the fundamental and first-overtone radial modes and their coupled terms. New observations and analyses support the most recent results of Pócs & Szeidl (2001). The fundamental period was essentially constant over the past 27 years with its standard value of  $0^{\text{d}}086017066$  ( $f_0 = 11.625600 \text{ cd}^{-1}$ ), while the first overtone period decreased at a rate of  $\frac{1}{P_1} \frac{dP_1}{dt} = -4.3 \times 10^{-8} \text{ yr}^{-1}$ . The amplitude variations in the two modes of AE UMa are detected at the milli-magnitude level on a time-scale of years. It seems that the amplitudes vary in opposite phases, implying an energy conservation or some kind of intrinsic variability cause. We deny the over-interpretation of the period change given by Hintz et al. (1997) and explore its reason.

**Key words.** stars: variable:  $\delta$  Scuti stars – stars: oscillations – stars: individual: AE UMa

## 1. Introduction

The star AE Ursae Majoris (= HIP 47181,  $\alpha_{2000} = 09^{\text{h}}36^{\text{m}}53^{\text{s}}$ ,  $\delta_{2000} = 44^{\circ}04'01''$ ,  $V = 11^{\text{m}}27$ ,  $P_0 = 0^{\text{d}}0860$ ,  $\Delta V = 0^{\text{m}}10$ , A9) (Rodríguez et al. 2000) has a longer observational history (of 45 years) since it was discovered to be a variable (Geyer et al. 1955; Tsevevich 1956; Filatov 1960). It was then classified as a dwarf Cepheid by Tsevevich (1973) and extensive studies were contributed by several authors (Szeidl 1974; Broglia & Conconi 1975; Braune et al. 1979; Rodríguez et al. 1992; Hintz et al. 1997). Previous studies unambiguously and consistently show that the light variations of AE UMa resulting in a remarkable beat phenomenon results from interference of two pulsations at  $f_0 = 11.6252$  and  $f_1 = 15.0308 \text{ cd}^{-1}$ , which correspond to the fundamental and first radial overtone modes, respectively. The period ratio  $P_1/P_0$  has remained constant at 0.773. By using a  $\delta m_1$  calibration for metal abundance, Rodríguez et al. (1992) obtained  $[M/H] = -0.3$  from the  $\delta m_1$  at minimum light. Hintz et al. (1997) derived the metallicity to be  $-0.1$  or  $-0.4$  (i.e.  $Z = 0.016$  or  $0.008$ ), from  $P_1/P_0$  and other calibrations, respectively. Based on the period ratio, metal abundance and other derived atmospheric parameters and photometric properties, AE UMa was re-classified as a normal Population I double-mode dwarf Cepheid or a high-amplitude  $\delta$  Scuti star (HADS) and it is not a

metal-poor Pop. II SX Phoenicis star as it is currently classified in the GCVS (Cox et al. 1979; Rodríguez et al. 1992; Hintz et al. 1997). At the same time, AE UMa is suggested to be in a main sequence (MS) stage or close to the end of the MS stage of evolution concerning the mass ( $1.85 \pm 0.1 M_{\odot}$ ) and age ( $1.1 \pm 0.1 \text{ Gyr}$ ) determined with the standard evolutionary tracks (Rodríguez et al. 1992).

In terms of a quadratic fit to O–C residuals, Hintz et al. (1997) showed that the fundamental period of AE UMa is continuously decreasing at a constant rate of  $-1.14 \times 10^{-10} \text{ d}^{-1}$ , which led to the citation of  $-48 \times 10^{-8} \text{ yr}^{-1}$  in Breger & Pamyatnykh (1998). If this value is real, it would be the fastest measured period change among the Pop. I radially pulsating  $\delta$  Scuti stars. The rate of the period decrease is similar to that expected for pre-MS variables. However, there exists no evidence that AE UMa is a pre-MS star (Breger & Pamyatnykh 1998). Moreover, the most recent work of Pócs & Szeidl (2001) contradicted this period change. According to these authors' investigations, the fundamental oscillation is very stable and the rate of its change is less than the error of its determination, i.e. smaller than  $10^{-12} \text{ d}^{-1}$ . The observed period change is hence reconciled with the theoretical expectation (Breger & Pamyatnykh 1998). Consequently, Pócs & Szeidl (2001) also suggest that AE UMa is in the post-MS evolutionary state. On the other hand, they point out that the first overtone period is definitely decreasing with a rate of  $(1/P_1)(dP_1/dt) = -2.0 \times 10^{-10} \text{ d}^{-1}$ .

This work aims to investigate the secular variations of the pulsation periods and their amplitudes on the

\* Figure 5 is only available in electronic form at <http://www.edpsciences.org>

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extended data base. In this sense, new observations were carried out in two observing seasons 2000 and 2001. In Sect. 3, we analysed the new data together with those available in the literature. We show further evidence supporting the explanations of Pócs & Szeidl (2001). Our results are discussed and summarized in Sect. 4.

## 2. Observations

As part of an ongoing program on the poorly studied  $\delta$  Scuti stars, AE UMa was observed from 4 March 2000 to 1 February 2001. In 2000, the photometry of AE UMa was performed with the light-curve survey CCD photometer mounted on the 85-cm telescope at the Xinglong Station of the Beijing Astronomical Observatory (BAO) of China. The photometer used a red-sensitive Thomson TH7882 576 $\times$ 384 CCD with a whole imaging size of 13.25  $\times$  8.83 mm<sup>2</sup> corresponding to a sky field-of-view of 11'.5  $\times$  7'.7, which allows sufficient stars to be toggled in a frame as reference. The magnitude differences between reference stars observed in the field of AE UMa yielded a typical accuracy of 0<sup>m</sup>.010 to 0<sup>m</sup>.006 during the observations. Three of the reference stars were detected as non-variables within the observational error and they were then used as comparisons to produce differential magnitudes for the variable as a mean combination as AE UMa - (C1+C2+C3)/3. Our finder chart is similar to Fig. 5 of Hintz et al. (1997), where the stars numbered 1, 2 and 4 correspond to our C1, C2 and C3, respectively. Their identifications are given in Table 2. The detailed technical features of the CCD system have been described in Wei et al. (1990). The procedures of data reduction including on-line bias subtraction, dark reduction and flatfield correction, are outlined in Zhou et al. (2001a).

In addition, the star was observed photoelectrically with the three-channel photometer designed for the Whole Earth Telescope campaign (Nather et al. 1990; Jiang & Hu 1998) on the 85-cm telescope at the BAO in 2001. C3 was chosen as the comparison, which was independently compared to C1 on one night. The standard deviation of the differential magnitudes between C3 and C1 yielded a general observational accuracy of 0<sup>m</sup>.005. The time-series data were collected through a standard Johnson *V* filter in both seasons. Exposure times were 60 s throughout the first run and 30 s for the second. Differential atmospheric and colour extinction effects were corrected whenever present in the light curves of AE UMa. In total, we obtained 6526 measurements consisting of 13 nights (89.6 hours) spanning 335 days. A journal of the observations is given in Table 1.

## 3. Data analysis

For the detection of time-evolution of the light variations of AE UMa, we have made use of the time-series data and times of maximum light available in the literature. We divided the data into six sets according to different sources

**Table 1.** Journal of Johnson *V* photometry of AE UMa.

Date	HJD(start) 2451000+	Points
2000.03.04	601.2754	441
2000.03.05	602.2747	506
2000.03.06	603.2786	199
2000.03.07	604.2634	269
2000.03.08	611.2344	475
2000.03.09	615.3259	453
2000.03.11	616.2881	370
2000.03.12	616.2881	373
2001.01.19	929.9221	578
2001.01.20	930.1720	703
2001.01.21	931.1718	618
2001.01.31	941.1405	754
2001.02.01	942.1410	817

**Table 2.** Comparison stars used in the photometry of AE UMa.

Stars	$\alpha$ (2000)	$\delta$ (2000)	<i>V</i>
C1 = GSC 2998-1166	09 <sup>h</sup> 37 <sup>m</sup> 12 <sup>s</sup> .06	43°58'20".9	11 <sup>m</sup> .7
C2 = GSC 2998-0963	09 <sup>h</sup> 37 <sup>m</sup> 17 <sup>s</sup> .27	43°56'39".6	11 <sup>m</sup> .7
C3 = GSC 2998-1249	09 <sup>h</sup> 37 <sup>m</sup> 28 <sup>s</sup> .57	44°01'17".4	11 <sup>m</sup> .1

**Table 3.** Data sets analysed for AE UMa. BC: Broglia & Conconi (1975), PS: Pócs & Szeidl (2001), RRL: Rodríguez et al. (1992).

Year	Nights	Filter	Measurements	Source
1974	7	<i>V</i>	973	BC
1974-77	20	<i>V</i>	591	PS
1981-86	9	<i>V</i>	549	PS
1987	8	<i>y</i>	229	RRL
1996-98	12	<i>V</i>	796	PS
2000-01	13	<i>V</i>	6526	present work

and time intervals: 1974, 1974-77, 1981-86, 1987, 1996-98, and 2000-01. They are described in Table 3. Given the known biperiodicity of AE UMa, the two frequencies  $f_1 = 11.62562$  and  $f_2 = 15.03149$   $\text{cd}^{-1}$  ( $P_0 = 0.086016883$  and  $P_1 = 0.0066527$  days) of Broglia & Conconi (1975) corresponding to the fundamental and first-overtone radial pulsation modes were adopted to describe the pulsational behaviour of the variable. At the same time, these two frequencies would be nonlinearly coupled so that combination frequencies  $if_0 \pm jf_1$  ( $i, j$  integer) should also be present in the light variations of the star. Accordingly, we carried out least-squares fits to the data of 12 frequencies involved in a complete fifth-order doubly periodic expansion (order =  $|i| + |j|$ ) (Fitch & Szeidl 1976). The 12 frequencies are identical to those used in Rodríguez et al. (1992). In fact, we found all the 12 terms and

**Table 4.** Results of Fourier analyses for the data sets of AE UMa in four different intervals. Frequencies are given in cycles per day. Fitting errors are only given for the two independent terms (0 refers to an error less than  $0.000004 \text{ cd}^{-1}$ ).

Terms	1974-77	1981-87	1996-98	2000-01
$f_0$	11.62557	11.62290	11.62560	11.62561
	1	0	2	0
$2f_0$	23.25123	23.25120	23.25118	23.25121
$f_1$	15.03097	15.07259	15.03122	15.03119
	3	1	5	2
$3f_0$	34.87671	34.87949	34.87678	26.65989
$f_0 + f_1$	26.65689	26.65680	26.65685	34.87685
$f_1 - f_0$	3.40283	3.40558	2.40565	3.40886
$2f_0 + f_1$	38.28220	39.28499	38.28239	38.28239
$4f_0$	46.50232	46.49970	46.50246	46.50553
$3f_0 + f_1$	49.90779	—	48.90809	49.90807
$2f_0 - f_1$	8.21727	—	—	8.21717
$5f_0$	58.12798	—	58.05347	58.12800
$4f_0 + f_1$	61.53349	—	—	61.53381

the sixth-order harmonic ( $6f_0$ ) when Fourier-reanalysing the data of Broglia & Conconi (1975). The procedures of 12-frequency fits, allowing amplitude and phase variations, together with Fourier-analysis were performed for each of the data sets and their bins. When the data had been whitened for these variations and a new amplitude spectrum calculated, we detected no additional frequencies as being intrinsic to the variable. The frequency analysis was performed using PERIOD98 (Breger 1990; Sperl 1998) and MFA (Hao 1991; Liu 1995). We refer the reader to Zhou et al. (1999, 2001b) for the procedure. The solutions for the six sets of data and their bins are highly consistent. From the combined data from 1974 to 2001, the two intrinsic frequencies were refined to be  $f_0 = 11.625600 \pm 0.0000002$  and  $f_1 = 15.031200 \pm 0.0000010 \text{ cd}^{-1}$  with amplitudes  $210.6 \pm 1.1$  and  $41.9 \pm 1.1 \text{ mmag}$ , respectively. The power spectra from this bin data did not show any trace of spurious peaks caused by amplitude and/or frequency modulations. In this regard, we understand that the pulsation periods and amplitudes of AE UMa are basically stable.

Given the root-mean-square deviations of observational noise corresponding to each data set, the errors on the fitted frequencies and amplitudes can be estimated following the formulae of Montgomery & O'Donoghue (1999). The results of the frequency analysis are given in Table 4. The new differential light curves along with the fit of the 12 frequency terms are presented in Fig. 5 (on-line).

### 3.1. Amplitude variation

To investigate the variability in amplitude, we have performed nonlinear least-squares fits to the new data and

to the data sets of Broglia & Conconi (1975), Rodríguez et al. (1992) and Pócs & Szeidl (2001) separately. We fit the light curves with the two frequencies  $f_1 = 11.62562$  and  $f_2 = 15.03149 \text{ cd}^{-1}$  and their interaction terms allowing variations of amplitudes and phases. The results, presented in Table 5 and Fig. 2, show that the amplitudes of the fundamental frequency  $f_0$  and first-overtone  $f_1$  were slightly variable from 1974 to 2001. Some interaction terms underwent obvious amplitude changes. In drawing Fig. 2, we have assumed the mean amplitude for data set 1996-98 as the value in 1998 because most of the data came from this year. The standard error of 5.5 mmag for the amplitude given in Hintz et al. (1997) might be underestimated. In practice, their amplitudes for both the two frequencies are much higher than any other values. They obtained about 12 hour time-series data on three separate nights covering 10 maxima. As a result, this might cause a higher uncertainty on their Fourier amplitudes. By ignoring the amplitudes in 1997, the variations in amplitudes were changing at a level of several millimagnitudes (about 5 mmag). Furthermore, there seems to be an opposite phase for the changing of amplitudes between the fundamental and first-overtone frequencies. That is, the amplitude of  $f_0$  decreased when that of  $f_1$  increased from 1974 to 2001. To summarize, the amplitudes of both the fundamental and first-overtone frequencies are clearly marked and are distinctive.

### 3.2. Period changes

Taking no account of the value from the 1981-87 data set, the results in Table 4 demonstrate the constant fundamental frequency ( $f_0$ ). The first overtone ( $f_1$ ) had a change up to  $0.0002 \text{ cd}^{-1}$ . In addition to the Fourier analyses, we also used the classic O-C method to examine the long-term period changes. From the present work, 45 new times of maxima are measured and tabulated in Table 6. Considering the collections of Hintz et al. (1997) and Pócs & Szeidl (2001) and discarding the earlier photographic and visual maxima which involved larger errors (rms = 0.024, according to a linear fit made by Hintz et al. 1997), we have a total of 164 times of maxima. By adopting the ephemeris of

$$\text{HJD}_{\text{max}} = 2442062.5823 + 0.086016883 E \quad (1)$$

(Broglia & Conconi 1975), O-C residuals and the cycles elapsed from this initial epoch were calculated. To fit the O-C residuals, we used a weighted least-squares fit technique. According to the quality of the observed maxima, they were assigned with different weights. Then we improve the light elements to

$$\text{HJD}_{\text{max}} = 2442062.58219 + 0.0860170697 E \quad (2)$$

$$\pm 0.00030 \pm 0.0000000038$$

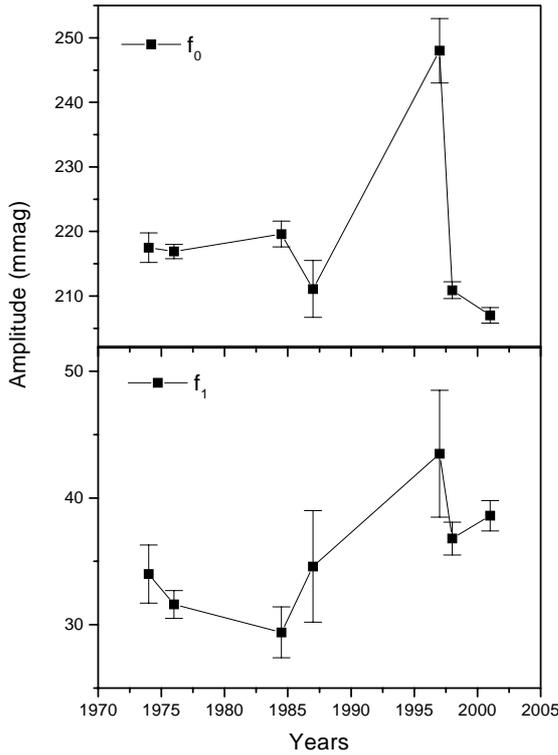
or

$$\text{O-C} = -0.0001062 + 1.86744 \times 10^{-7} E \quad (3)$$

$$\pm 0.0002987 \pm 0.0379$$

**Table 5.** Amplitude variability of AE UMA determined by Fourier analysis, based on the data sets in Table 3, assuming  $f_0 = 11.62562$  and  $f_1 = 15.03149$   $\text{cd}^{-1}$ . The results for the “1997” column are adopted from Hintz et al. (1997).

Frequency ( $\text{cd}^{-1}$ )	Amplitude in milli-magnitudes						
	1974	1974-77	1981-86	1987	1996-98	1997	2000-01
$f_0 = 11.62562$	217.5	216.9	219.6	211.1	210.9	248.0	207.0
$2f_0 = 23.25124$	74.0	71.8	75.2	70.7	69.6	75.0	69.9
$f_1 = 15.03149$	34.1	31.6	29.4	34.6	36.8	43.5	38.6
$3f_0 = 34.87688$	28.1	26.7	32.6	28.1	28.5	36.0	26.4
$f_0 + f_1 = 26.65711$	23.6	17.3	14.6	23.9	24.1	20.0	25.6
$f_1 - f_0 = 3.40587$	19.1	17.2	14.0	23.9	20.2	15.0	23.6
$2f_0 + f_1 = 38.28273$	13.0	11.2	7.0	11.7	12.0	12.0	13.8
$4f_0 = 46.50248$	12.6	10.9	16.9	12.3	11.1	18.0	12.4
$3f_0 + f_1 = 49.90835$	7.8	3.2	3.7	5.8	7.5	6.0	6.7
$2f_0 - f_1 = 8.21975$	5.4	13.3	8.6	5.1	1.5	8.0	9.9
$5f_0 = 58.12810$	6.9	8.2	14.4	7.6	3.9	7.0	6.3
$4f_0 + f_1 = 61.53397$	4.5	5.6	5.2	5.1	2.3	0.0	4.6
std errors of ampl. (mmag)	2.3	1.1	2.0	4.4	1.3	5.0	1.2

**Fig. 2.** Amplitude variability of AE UMA from 1974 through 2001. Note the great uncertainty in 1997, see Sect. 3.1 for an explanation.

with a standard deviation of  $\sigma = 0.00226$  through a linear fit. Accounting for the evolution-origin continuous changing of period in AE UMA, we obtained a forced quadratic solution using a second-order polynomial fit:  $\text{HJD}_{\text{max}} =$

$$2416564.58220 + 0.0860170693 E - 4.0809 \times 10^{-15} E^2 \pm 0.00034 \pm 0.000000186 \pm 159.0 \quad (4)$$

or O-C =

$$-0.000101948 + 1.8628 \times 10^{-7} E - 4.0809 \times 10^{-15} E^2 \pm 0.000342665 \pm 0.1857 \pm 159.0. \quad (5)$$

However, the  $\sigma$  value did not change, which indicates no improvement on the previous one. If the same ephemeris

$$\text{HJD}_{\text{max}} = 2442062.5824 + 0.08601707 E \quad (6)$$

used by Pócs & Szeidl (2001) was adopted, then we have

$$\text{O-C} = -0.000135243 + 9.78611 \times 10^{-10} E \pm 0.000297279 \pm 3.7722 \quad (7)$$

( $\sigma = 0.00225$ ) which manifests the O-C residuals as scattered points around the zero axis in the O-C diagram (see the bottom panel of Fig. 3). Therefore, the period in Eq. (6) is a more appropriate value, while that in Eq. (1) is slightly underestimated. The quadratic term of Eq. (5) suffered from great uncertainty. At best, this term indicates a period change rate of  $\frac{1}{P_0} \frac{dP_0}{dt} = -8.1618 \times 10^{-15}$  days per cycle, i.e.  $-9.49 \times 10^{-14} \text{d}^{-1}$  ( $= -0.35 \times 10^{-10} \text{yr}^{-1}$ ), which conforms to the results of Pócs & Szeidl (2001) and hence confirms that the changing rate of  $-1.14 \times 10^{-10} \text{d}^{-1}$  (or  $-42 \times 10^{-8} \text{yr}^{-1}$ ) determined by Hintz et al. (1997) is incorrect. Indeed, Breger & Pamyatnykh (1998) had a suspicion about this period change, as we mentioned in the introduction. In the last paragraph of this section, we explored the reason for their over-interpretation of the O-C values. An up-to-date diagram for the differences between the observed and calculated times of maximum light are plotted versus the cycles in Fig. 3. It is now very clear that the temporal dependence of the main frequency of AE UMA is not distinct. In general, the fundamental frequency is constant from 1974 to 2001.

Given the star's double-mode nature, the observed times of maximum light we have used are in fact the result of the interaction of the two modes and are not the real times of maxima for the fundamental period itself. Therefore, it is not ideal to use Eq. (1) to calculate the times of maxima for the fundamental period alone. In principle, we should first remove the effect of the first overtone period. That is, to prewhiten the data with  $P_1$ , then to find out the times of maxima from the residuals. In this way, one obtains the practically observed times of maxima for the fundamental period. Rodríguez et al. (1997b) used this procedure to analyse the period changes of AN Lyn. However, it is not practical to apply this to AE UMa owing to the unavailability of time-series data on which some of the times of maxima were based. On the other hand, because of the great difference between the oscillation amplitudes of the two periods, the analysis we made above is basically reliable to reveal the behaviour of the main period. Alternatively, Pócs & Szeidl (2001) used a new formula considering the changes of the first overtone period as well as the phase modulation amplitude. In similar but simpler manner, we can directly replace the period in Eq. (1) with the strong beat period  $P_m = 0^d293616$  to study its secular changes. The initial time of maximum light was fixed. We used the ephemeris

$$\text{HJD}_{\max}(P_m) = 2442062.5823 + 0.293616 E \quad (8)$$

to establish the O–C diagram for  $P_m$ . Then, we obtained

$$\text{O–C}(P_m) = -0.000062 + 3.10528 \times 10^{-7} E \quad (9)$$

$$\pm 0.01171 \pm 5.07077$$

( $\sigma = 0.08849$ ) and O–C( $P_m$ ) =

$$-0.00698 + 2.89257 \times 10^{-6} E - 7.73004 \times 10^{-11} E^2 \quad (10)$$

$$\pm 0.01338 \pm 2.47575 \quad \pm 7.25483$$

( $\sigma = 0.08845$ ). Thus we found  $\frac{dP_m}{dt} = -1.546 \times 10^{-10} \text{ d cycle}^{-1}$ , i.e.  $\frac{1}{P_m} \frac{dP_m}{dt} = -5.265 \times 10^{-10} \text{ d}^{-1}$ . So from  $f_1 = f_m + f_0$  and ignoring the term  $\frac{1}{P_0} \frac{dP_0}{dt}$ , we arrived at  $\frac{1}{P_1} \frac{dP_1}{dt} = -4.3 \times 10^{-8} \text{ yr}^{-1}$ . This value agrees with the rate of change of  $-7.3 \times 10^{-8} \text{ yr}^{-1}$  derived by Pócs & Szeidl (2001). We also applied the analysis to  $P_1 = 0^d066527$  and we obtained

$$\text{O–C}(P_1) = -0.00167 + 2.07548 \times 10^{-8} E \quad (11)$$

$$\pm 0.00260 \pm 2.55015$$

( $\sigma = 0.01964$ ) and O–C( $P_1$ ) =

$$-0.00138 - 3.81685 \times 10^{-9} E + 1.66675 \times 10^{-13} E^2 \quad (12)$$

$$\pm 0.00298 \pm 124.931 \quad \pm 8.29485$$

( $\sigma = 0.0197$ ). The quadratic term of Eq. (12) indicates a rate of change  $\frac{1}{P_1} \frac{dP_1}{dt} = 1.8 \pm 9.1 \times 10^{-9} \text{ yr}^{-1}$ . The value itself is much less than the error of its determination. In the extreme case, however, this term gives a value in the same order ( $10^{-8} \text{ yr}^{-1}$ ) with the value derived from Eq. (10), which is relatively certain. By comparing the

**Table 6.** New times (HJD 2400000+ days) of maximum light of AE UMa.

Times(HJD)	Cycles	Times(HJD)	Cycles
51608.07163	110 972	51615.03414	110 053
51608.15774	110 973	51615.12604	110 054
51608.23952	110 974	51615.20980	110 055
51608.32640	110 975	51615.29194	110 056
51609.01858	110 983	51615.98547	110 064
51609.10062	110 984	51616.07047	110 065
51609.18650	110 985	51616.15263	110 066
51609.27701	110 986	51616.23885	110 067
51609.35828	110 987	51929.25560	114 706
51610.04498	110 995	51929.34641	114 707
51610.99689	111 006	51930.20585	114 717
51611.08211	110 007	51930.28851	114 718
51611.16270	110 008	51930.37206	114 719
51612.02457	110 018	51931.23155	114 729
51612.10905	110 019	51931.32029	114 730
51612.20104	110 020	51941.21021	114 845
51612.28463	110 021	51941.29794	114 846
51612.37043	110 022	51941.38811	114 847
51612.96921	110 029	51942.15625	114 856
51613.06089	110 030	51942.24730	114 857
51613.14535	110 031	51942.33107	114 858
51613.22764	110 032	51942.41414	114 859
51613.31556	110 033		

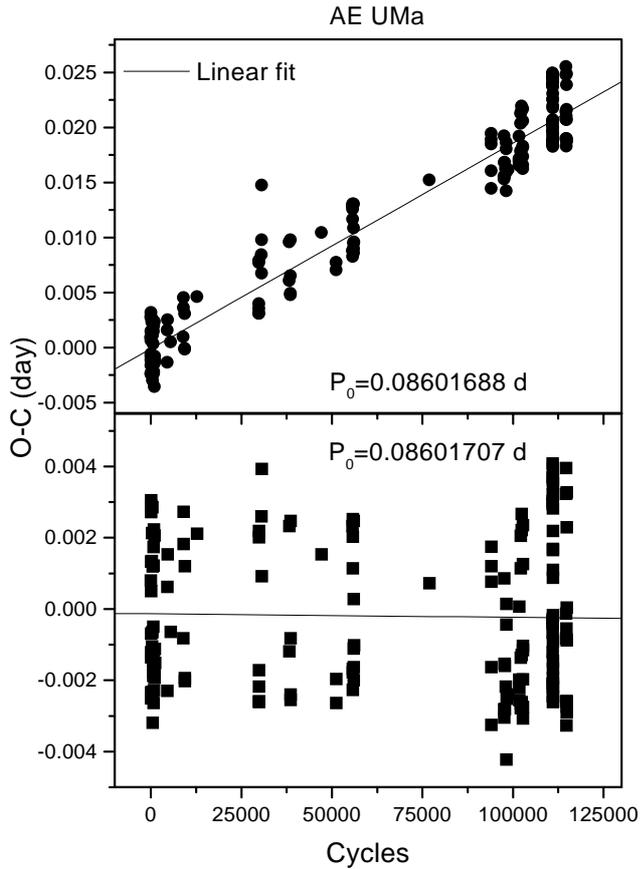
quadratic term of Eq. (12) with that of Eq. (5), we can see  $\frac{dP_1}{dt}$  is larger than  $|\frac{dP_0}{dt}|$ , i.e.  $P_1$  was changing (increasing) more obviously than  $P_0$ . The resulted O–C diagrams for  $P_m$  and  $P_1$  are displayed in Fig. 4.

In this paragraph, we tried to search for the reason leading Hintz et al. (1997) to their faster changing rate. All citations of figures, equations and table here refer to the original paper. First, we checked the times of maximum light listed in their Table 1. The entry “42886.496” with cycle number 9346 (from Braune et al. 1979) was misprinted and should be “42866.496”. Secondly, the caption of their Fig. 3 is not accurate. The diagram in fact presents all the data in their Table 1, not only for the photomultiplier and CCD data. Thirdly, these authors calculated the cycles using their Eq. (3), which was not clearly demonstrated to the reader. Fourth, we reanalysed all the 72 times in Table 1 using also Eq. (3). We did not take mean O–C values. We obtained  $\text{HJD}_{\max} =$

$$2442062.5812 + 0.08601581 E - 4.97394 \times 10^{-12} E^2 \quad (13)$$

$$\pm 0.0014 \pm 0.00000002 \quad \pm 0.26059$$

( $\sigma = 0.00994$ ) which agrees perfectly with their Eq. (4). If we reanalyse the photomultiplier and CCD data (those



**Fig. 3.** O–C diagrams for AE UMa, based on 164 times of maximum light available up to date. Upper: the period was underestimated; lower: an appropriate period was used.

times after zero cycle, 62 times) alone, then we obtained  $HJD_{\max} =$

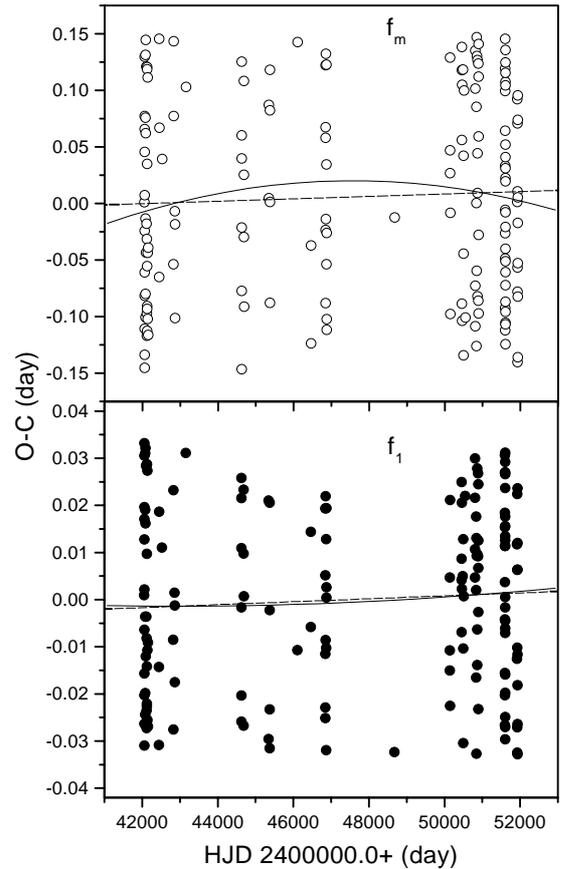
$$2442062.5827 + 0.08601585 E - 5.71034 \times 10^{-12} E^2 \quad (14)$$

$$\pm 0.0004 \pm 0.00000003 \pm 0.34421$$

( $\sigma = 0.00238$ ) which gives better fitting and larger period change than the former. It is clear now that the over-interpretation of these authors is due to the limited data, especially the inclusion of the earlier times. This could not be avoided in their analysis. However, the authors paid no attention to this point. Similarly, as mentioned above, their Fourier analysis was also heavily affected by their shorter data set (three nights). We should apply caution when analysing short-baseline data with the O–C and Fourier methods. Both the present contribution and the analysis of Pócs & Szeidl (2001) have put the earliest photographic and visual measures aside. These times are less reliable relative to the photoelectric and CCD ones.

#### 4. Conclusions and discussion

On the basis of the photometric data from 1974 to 2001, the secular behaviour of pulsation of the high-amplitude bimodal  $\delta$  Scuti star AE UMa was thoroughly investigated. The analysis of the binned data from 1974 to 2001



**Fig. 4.** O–C diagrams for AE UMa. Upper: the modulation period  $P_m = 0^d.293616$ ; lower: the first overtone period  $P_1 = 0^d.066527$ . The dash-lines for linear fits, while the solid lines for quadratic fits.

refined the two oscillation frequencies to  $f_0 = 11.625600 \pm 0.0000002$  and  $f_0 = 15.031200 \pm 0.0000010 \text{ cd}^{-1}$ . No additional frequencies can be identified to be real in the residual spectrum after removing the two known frequencies and their nonlinearly-coupled terms. The light variations of AE UMa can be perfectly reproduced with the fundamental and first-overtone radial modes and their interaction terms and harmonics up to sixth-order.

New observations and our analyses support the most recent results of Pócs & Szeidl (2001) and hence reject those of Hintz et al. (1997). The fundamental period was essentially constant over the years from 1974 to 2001 with its standard value of  $0^d.086017066$  ( $f_0 = 11.625600 \text{ cd}^{-1}$ ), while the first overtone period was decreasing at a rate of  $\frac{1}{P_1} \frac{dP_1}{dt} = -4.3 \times 10^{-8} \text{ yr}^{-1}$ . The reason for the over-interpretation of the change of the fundamental period (Hintz et al. 1997) is the limitation of data in their analysis.

The amplitude variations in the two modes of AE UMa are detected at the milli-magnitude level on a time-scale of years. This observed behaviour cannot be explained by the observational uncertainties, as shown by the error bars in Fig. 2. However, the change is not so prominent as that of AN Lyn, which annually shows significant linear increments (Rodríguez et al. 1997a; Zhou 2001), or as the

dramatic variations in 28 And (Rodríguez et al. 1998) and the variations accompanied with period jumps in V1162 Ori (Hintz et al. 1998; Arentoft & Sterken 1999).

It seems that the amplitudes of the two pulsation modes vary in opposite phases, implying energy conservation. Hence, the amplitude variations might be caused by some kind of intrinsic variability, such as a slow transfer of pulsation energy from one mode to another. Another reason for the amplitude variations is the interaction between modes, because there exists a strong periodic modulation (a beat period  $P_b = 0^m293616$ ) between the fundamental and first overtone modes. Moreover, the amplitude variability of the coupled frequencies,  $f_i + f_j$ , is more obvious than those of the two intrinsic ones. A similar case of this kind of opposite-phase amplitude variations occurred in the  $\delta$  Sct star DQ Cephei (Li & Fang 1999). No periodic feature can be seen from the present results regardless of the data obtained in 1997 from Hintz et al. (1997). This in turn supports an evolutionary origin. Nevertheless, if the values from these authors are reliable within the error of 5 mmag (see Table 5), a cyclic behaviour over several decades of the amplitude variations in the star would be possible, excluding the evolutionary origin.

The high-amplitude  $\delta$  Sct stars are often assumed to be constant in their amplitudes of light curves in the long term. A detailed analysis of the behaviour of the amplitude for monoprotic HADS was performed by Rodríguez (1999). The author analysed a sample of seven stars and concluded that long-term changes of amplitude of the light curves for any of these sampled stars are not significant. However, the long-term changes of amplitude, corresponding to one or several modes of a pulsating variable, are quite common among low-amplitude  $\delta$  Sct stars. A well-studied star for amplitude variability is 4 CVn, which shows strong amplitude variability with time-scales of ten years or longer. Another example of amplitude variations is  $\theta^2$  Tau (Li et al. 1997). However, for all cases, the real origin of amplitude variability in  $\delta$  Sct stars is presently not understood (Breger 2000). We have seen the amplitude variations, possibly accompanied by period changes in the HADS V1162 Ori (Arentoft & Sterken 1999), XX Cyg (Zhou et al. 2001c), the present star AE UMa and others, e.g. AI Vel (Walraven et al. 1992). From an observational point of view, amplitude variability of HADS, in particular, the double-mode HADS, deserves further investigation.

According to the basic pulsation equation  $Q = P\sqrt{\bar{\rho}}$ , the mean density and pulsation mass of the variable can be calculated in terms of its principal period and pulsation constant. With the photometric parameters  $M_{\text{bol}} = 1^m73 \pm 0.2$ ,  $\langle T_{\text{eff}} \rangle = 7560$  K,  $\langle \log g \rangle = 3.97$ , derived by Rodríguez et al. (1992), and the empirical formula

$$\log Q_i = -6.456 + \log P_i + 0.5 \log g + 0.1 M_{\text{bol}} + \log T_{\text{eff}} \quad (15)$$

given by Petersen & Jørgensen (1972), we derive the observed pulsation constants  $Q_0 = 0^d0329$  and  $Q_1 = 0^d0254$  for  $P_0 = 0^d086016883$  and  $P_1 = 0^d066527$ , respectively.

Then we find  $\bar{\rho}_* = 0.146 \bar{\rho}_\odot$ . There is a discrepancy between this value and 0.135 given by Fitch & Szeidl (1976). The latter resulted from the pulsation calculations for high-metal-content evolutionary models with  $X = 0.602$  and  $Z = 0.044$ . However, AE UMa is metal-poor,  $Z = 0.01$  if  $[\text{Fe}/\text{H}] = -0.3$  (Rodríguez et al. 1992). By means of the radiation law

$$\log R/R_\odot = -0.2 M_{\text{bol}} - 2 \log T_{\text{eff}} + 8.472, \quad (16)$$

we derived a radius of  $R = 2.34 R_\odot$  for AE UMa. Then we got a pulsation mass  $M_{\text{pul}} = 1.87 M_\odot$ , which is in good agreement with the evolutionary mass ( $1.8 M_\odot$ ) determined from theoretic models by Rodríguez et al. (1992). It is also consistent with the result from the relation

$$\log M/M_\odot = 12.502 + \log g - 0.4 M_{\text{bol}} - 4 \log T_{\text{eff}} \quad (17)$$

(Petersen & Jørgensen 1972). The radius and density support Eq. (5) of Viskum et al. (1998), which was derived for FG Vir, a well-studied multiperiodic  $\delta$  Sct star near the end of its MS evolution as AE UMa. The mass and radius were further checked against the recent biparametric calibrations of Ribas et al. (1997). As a consequence of the physical parameters, AE UMa is distinguished from the metal-deficient Pop. II SX Phe stars.

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